Comprehensive Theoretical Study on Threshold Power of Stimulated Brillouin Scattering in Single-Mode Fibers

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ABSTRACT— We have investigated and developed a theoretical approach to explore stimulated Brillouin scattering (SBS) phenomena in single mode fiber. SBS happening threshold power condition has been studied in terms of fiber parameters and input pump power. To assess threshold power precisely, the pump depletion effect and fiber loss has been included by employing 1% criterion. The threshold exponential gain G_{th} can be anticipated by this simulation which strongly depends on the fiber length Brillouin gain content and effective area. The value of G_{th} is not a constant as usually assumed in the literature and its value is 4 for the longer lengths and between 10 and 18 is for relatively shorter lengths. This simulation can anticipate the optimum length of fiber against the every launched pump power to generate SBS effect.

KEYWORDS: Stimulated Brillouin scattering, single mode fiber, exponential Brillouin gain.

I. INTRODUCTION

Brillouin scattering is a nonlinear optical process that can occur in optical fibers at a large intensity. Since the Brillouin fiber amplifiers (BFA) [1] structures has been implemented in a wide range of applications such as narrow band filtering [2], pulse propagation control in optical fibers [3], and distributed strain and temperature Brillouin sensors (DBS) [4], the study of this effect hasn't been ceased. For most of these applications especially in Distributed Brillouin sensors, the sensor parameters such as tens of kilometers of sensing lengths which depends on pump and Stokes power levels roles must be identified and clear to provide optimum sensing operations based on pump depletion along the fiber [5]. Therefore, to evaluate the power budget requirements, the achievable gain and saturation characteristics of Brillouin amplifiers, precise and quantitative treatment of the SBS process is necessary. The most common parameter, which characterizes SBS effect, is its threshold pump power. From practical convenience point of view, the threshold is defined to describe the pump power, P_{th}, at which the output Stokes power is a some measurable fraction, r, of P_{th}; r is then used as a threshold criterion [6]. The SBS threshold which is characterizing the SBS phenomena is defined by the exponential gain [7] where g is the SBS gain coefficient of the medium, L_{eff} is its effective length and A_{eff} is the effective cross sectional area of the interaction region. There are some problems with existing evaluations of the SBS threshold on fiber. Standard steady- state equations that describe SBS are well known [8]. The threshold condition of SBS effect has been explored theoretically and experimentally as reported in papers [9-12]. It is common to ignore the effects of pump depletion or attenuation in analyzing SBS[10, 6, and 11] and G_{th} is assumed constant, of value of nearly 21 [9, 10] or with modern optical fibers this factor is replaced by 19 [12]. However, this value is dependent on four significant parameters as effective length, effective area, input pump power and Brillouin gain coefficient of the used fiber. This can be a poor assumption in single mode fibers, where the lengths are too long to neglect such effects.

Recently, it has been tried to describe a physically consistent theoretical treatment of G_{th} by considering all of the effective parameters except the fiber loss [12] based on Tong interpretation [8].Although the numerical explore can predict that G_{th} depends on some mentioned main factors, they didn't discuss the precise sensitivity of this factor and its exact value by incorporating different features of used fiber.

In this paper, we have developed the numerical solutions of the SBS equation based on Bayvel [12] normalization method to evaluate G_{th} dependence on three main parameters as fiber length, effective area, and Brillouin gain coefficient. This study assumes the Stokes emission initiated by spontaneous Brillouin scattering, SpBS, and uses the 1% criterion for the threshold definition. Within this work, to study the accurate contribution of the fiber optic parameters in threshold condition, transmitted pump and Stokes power were precisely reviewed with more detail and then, compared with the experimental and simulation results to verify this exploration. This analysis results in a meaningful consideration in the exact trend and value of G_{th} around 21 obtained by Smith [13] which is a very rough constant estimation for all common and typical sort of single mode fibers.

II. THEORY

SBS is a nonlinear optical phenomenon which involves an interaction between an incident wave, pump signal, a scattered wave, and an acoustic wave (phonon). For single mode fiber of length L, the differential equations describing the evolution of the pump and Stokes waves with position along the fiber z can be written as by [10]:

$$\frac{dI_S}{dz} = -g_B I_P I_S + \alpha_S I_S \tag{1}$$

$$\frac{dI_P}{dz} = -\frac{\omega_P}{\omega_S} g_B I_P I_S - \alpha_P I_P \tag{2}$$

where g_B is the Brillouin gain coefficient, I_P, I_S are the pump and Stokes wave intensities, the absorption coefficients α_P and α_S account for fiber losses at pump and Stokes frequency which are shown by ω_P and ω_S, respectively. Eq. (1) is obtained using the fiber losses due to Stokes wave determined by fiber loss coefficient at Stokes frequency and counter propagating nature of this wave. The feedback process responsible for Brillouin scattering is controlled by these two coupled equations, (1) and (2).

Two simplifying assumptions can be made in the equations considering the fact that the Brillouin shift is relatively small, first is $\omega_p = \omega_s$ which is followed by second that is $\alpha_p \approx \alpha_s \equiv \alpha$. The development of SBS in optical fibers then can be governed by set of the two coupled modified equations as follows [2]:

$$\frac{dI_S}{dz} = -g_B I_P I_S + \alpha I_S \tag{3}$$

$$\frac{dI_P}{dz} = -g_B I_P I_S - \alpha I_P \tag{4}$$

Solutions of the obtained set of coupled equations in steady state describe the interaction between the early mentioned involving phenomena. The first terms of Eq. (3) and (4) represents the Brillion gain and the pump depletion, respectively. In case where the Stokes power is much smaller than the pump power, one can assume that pump power is not depleted and therefore the term of "-$g_B I_P I_S$" in Eq. (4) can be neglected [10]. It leads to more familiar closed-form expression of the SBS threshold power for fibers as follow [10]:

$$P_{th} \approx 21 \frac{A_{eff}}{g_B^{(0)} L_{eff}} \tag{5}$$

where $L_{eff} = (\mathbf{1} - \exp(-\alpha L))/\alpha$ is the effective length of interaction. It is based on the absence of pump depletion that the reflected Stokes optical power at the beginning of the fiber (z=0) is equal to the input pump power) ($P_{reflected} = P_{in}$) [8, 12]. These conditions are not experimentally apparent as the pump depletion unavoidably happens and consequently the backscattered Stokes power becomes less than the pump power. Considering the latest facts, the calculation of the threshold condition was revised, by revisiting Eqs. (3) and (4). Fibers with different lengths, Brillouin gain coefficient, and loss coefficient produce different behaviors in making comparison and generalization of the results of analyses that can proved to be complicated. Practically the SBS threshold power should be defined as the amount of input power producing a reflected power equals to a fraction, r, of the pump ($P_{reflected} = r \times P_{in}$). This fraction factor can vary between $\approx 10^{-4}$ and $\approx 10^{-1}$ according to [7, 14]. Furthermore in accurate determination of the threshold condition the influence of pump depletion term should be considered.

Considering the latest facts calculation of the threshold condition was revised returning back to the equations 3 and 4. Fibers with different length, Brillouin gain coefficient, and loss coefficient produces different behavior making comparison and generalization of the results of analyses very complicated. Therefore, these equations are normalized by first defining a new normalized spatial variable, ζ so that $\zeta = z/L$ with $0 \le \zeta \le 1$. Based on the input pump intensity, I_0, at $\zeta = 0$ normalized intensities, P and S, were defined as follows [8]:

$$P(\zeta) = \frac{I}{I_0} = P \tag{6}$$

$$S(\zeta) = \frac{I}{I_0} = S \tag{7}$$

where $P(0)=1$ is the normalized input pump intensity. The pump and Stokes equations can be rewritten as:

$$\frac{dP}{d\zeta} = -GSP - \beta P \tag{8}$$

$$\frac{dS}{d\zeta} = -GSP + \beta S \tag{9}$$

where $G = g_B L I_0$ is the gain factor, and $\beta = \alpha_0 L$ is the loss. It is worth noting that the boundary conditions, which are known as the launched pump and Stokes intensities, are specified at the opposite ends of the fiber as $P(0)=1$, and $S(1)=10^{-9}$. So, the algorithm adopted in solving the equations with these boundary conditions is a precise model based on the fourth order Runge-Kutta algorithm using 10^{-4} accuracy.

III. Normalized Condition Results

The results of the numerical solution of the coupled (8) and (9) equations under the mentioned boundary conditions are studied against the fiber length for different fiber loss. In the case of lower values of loss (especially the curves $\beta=0, 1, 2$), Fig. 1(a) indicates the significant and strong effect of the pump depletion by the Stokes intensity which is more effective for $\zeta > 0.2$. The pump intensity trend is following the exponential decay $exp[-\alpha_0 L]$ caused by the fiber loss for higher values of loss (curves $\beta=5, 8$).

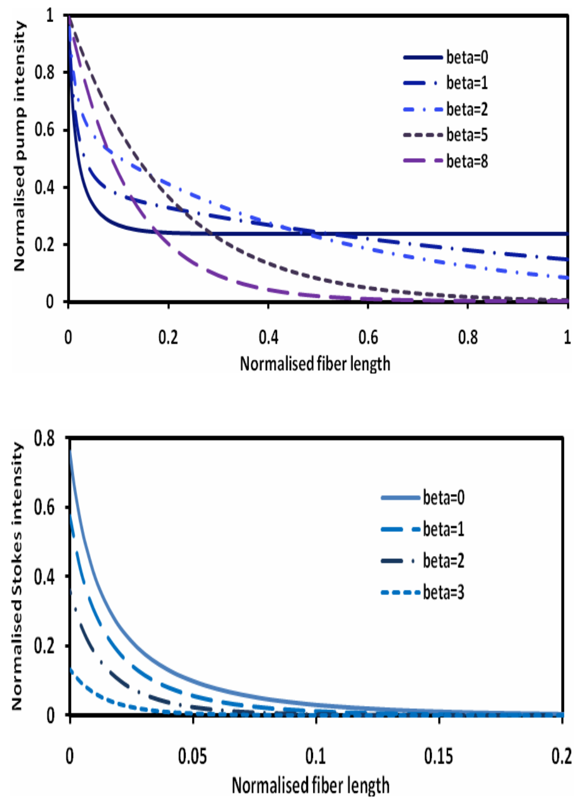


Fig. 1. Normalized transmitted pump and Stokes intensities vs. fiber length for different fiber loss.

Spontaneous Brillouin scattering is initiating and amplifying from noise starting at ζ=1 which is the end fiber, $z=L$. In another word, the value specified for the Stokes intensity boundary condition is the Stokes noise which creates and guides the Stokes at the other side (input end) of fiber. Therefore, according to the counter propagating nature of the Stokes intensity, the remarkable content of Stokes intensity is achievable at the end of fiber, for $0 < \zeta < 0.2$, as shown in Fig. 1 (b). Although the value of the Stokes initiating noise can be a source of the error, its caused changes is less than 10% in the outcomes of the $P(1)$ and $S(0)$. Since for the higher values of fiber loss $\beta >3$ the input pump power is below threshold and it cannot provide considerable Stokes intensity such that only the low loss curves can be illustrated as shown in Fig. 1 (b).

IV. Comparison of Numerical and Experimental Results

To assess the implications of the obtained results of these calculations and simulation for practical systems, we have also experimentally explored the outputs using the setup as shown in Fig. 2. This configuration is the most common and simplest of means to quantify the SBS threshold. In this setup, a tunable laser source (TLS) is used as the Brillouin pump (BP) or a narrow line width laser with maximum peak power approximately 5 dBm at the wavelength 1550 nm and the BP wide line width 15 MHz. The amplified BP power by Erbium doped fiber is launched into the free end single-mode fiber (SMF) through the ports 1 and 2 of the optical circulator. The back reflected signal is detected through the port 3 of the OC by using the optical spectrum analyzer (OSA) with the resolution 0.01 nm.

The under test fiber is a single-mode fiber (SMF) with an effective area A_{eff} of 80 µm^2 and with an attenuation coefficient of 0.2 dB/km or α (km^{-1})=α(dB/km)×(ln(10)/10)= 0.04605 km^{-1}. Since there is no Stokes input at the end of the test fiber, the obtained Brillouin scattering is initiated from thermal fluctuations in the density of the fiber core [14]. To explore the numerical results for a given length of the single mode fiber, L, we first assume that a pump wave with intensity of $I_p(0)$ is launched into the fiber. We also assume room temperature, $T=293\ K$, a phonon lifetime of $T_B=1/\Gamma_a=7ns$, and an acoustic frequency of $\Omega/2\pi$=11 GHz. Also, for arbitrary polarization, the Brillouin gain coefficient is reduced by a factor of 1.5 [14], so we have:

$$g_B = \frac{1}{1.5}\left(\frac{\gamma_e^2\omega^2}{\rho_0 n c^3 \nu \Gamma_a}\right) \qquad (10)$$

where n= 1.447, c=299.8 m/µs, and ν = 5960 m/s, and $\rho_0 = 2210$ kg/m^3 the gain coefficient is $g_B=1.091\times10^{-11}$ m/W. The electrostrictive constant γ_e can be evaluated by $\gamma_e=n^4(p_{11}+2p_{12})/3$ which is defined based on the elasto-optic coefficients, p_{11}=0.113, p_{12}=0.252 [13].

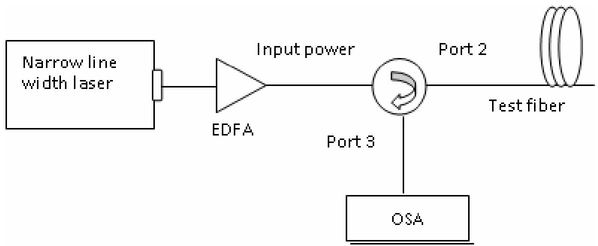


Fig. 2. The experimental configuration to measure SBS power threshold.

Based on these parameters, we iteratively solve (18) over a range of values of the launched pump power $I_P(0)$ to determine the reflected Stokes intensity and its regarding conversion efficiency.

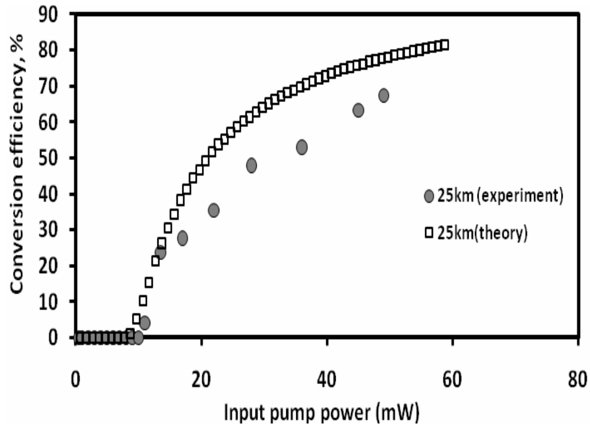


Fig. 3. Conversion efficiency obtained by the theoretical and experimental results Vs. input pump power.

Fig. 3 illustrates the obtained numerical and experimental conversion efficiency percentage, $(I_S(0)/I_p(0))\times 100$, as a function of the incident pump power $Pp(0)$ for 25 km long single mode fiber with the mentioned characteristics. In practice, the SBS threshold is likely to be higher due to the other loss sources in the experimental configuration. A good agreement can be observed between the experimental results and the numerical predictions used.

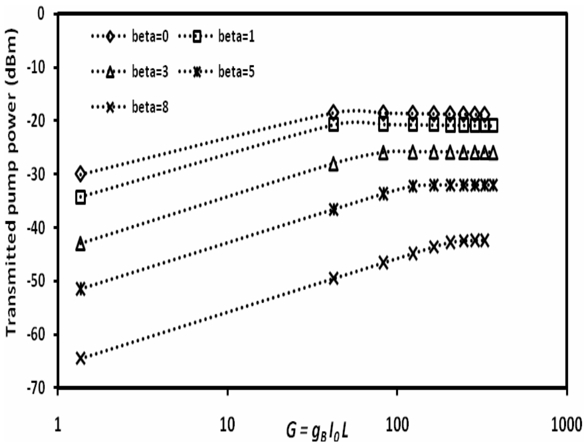


Fig. 4. Transmitted pump power vs. gain $G=g_BI_0L$ for different fiber loss.

Based on the mentioned parameters for a typical SMF, the variation of transmitted pump power with Brillouin gain $g_B\ I_0\ L$ was plotted corresponding to the pump power intensity variation for different fiber loss in Fig. 4. It can be seen that the transmitted pump intensity value monitored at the end of the fiber increases linearly to a constant value as a saturation plateau. These converting points (fringes) from the linear increment to the constant saturation provide a definition of threshold power regarding the input pump power [14]. As shown Fig. 4, transmitted power becomes depleted beyond this certain input pump power of threshold power which indicates the power transfer from pump signal to Stokes wave.

The pump to Stokes conversion efficiency increases as the values of fiber loss decrease up to even nearly *100%* for zero loss one as shown in Fig. 5. Beyond the threshold power, the sharp growth of Stokes intensity is located exactly with the value of *1%* conversion efficiency. Equivalent's exponential Brillouin gain , g_BI_0L is 18, 26, 41 for β=0, 1, 2, respectively such that as increasing the fiber loss and input pump power this value also is changing. To verify and compare these results with other simulation results, we consider an example of a 175km SMF with loss of 0.2dB/km (corresponding to α_0L equals 8) according to Ref. 8. The last curves in Fig. 4, and 5 show that for this case, the threshold occurs at value of g_BI_0L of nearly 177. For the peak value of g_B for fused silica of 4.6×10^{-11} m/W and effective area of 5×10^{-11} m^2, the threshold value is calculated by this simulation results around 1.1 mW. At threshold the back scattered Stokes power approximately -20 dBm, corresponding to 1% of the pump power which is in a good agreement with reported value in [8].

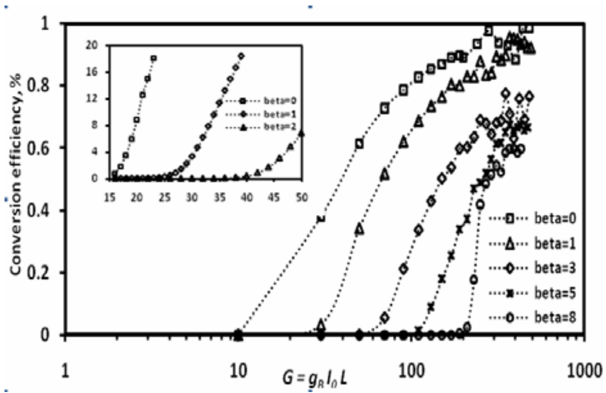
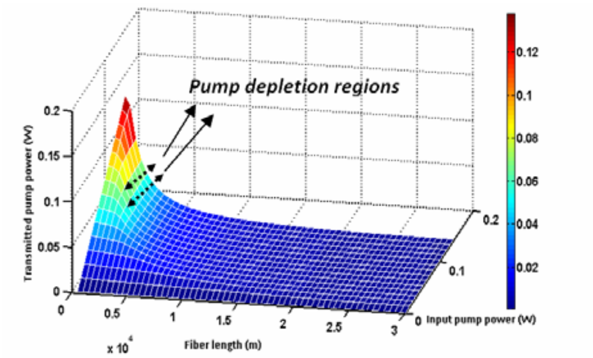


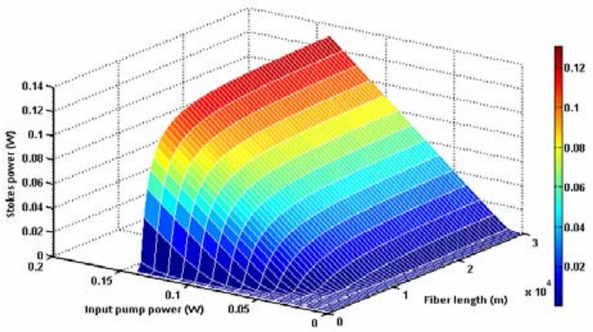
Fig. 5 Conversion efficiency vs. gain $G = g_B I_0 L$ for different fiber loss which in the inset graph the exact value of G can be more highlighted and clear with linear axis.

V. Threshold Pump Power Evaluation

In the previous sections, the spatial and input pump power dependence of the normalized transmitted pump and Stokes intensity were depicted in separate figures based on one by one calculation for different fiber loss. To further verify and progress these theoretical results, we have adopted and developed our algorithm to relate these two results in one figure. In other words, for any different length, the transmitted pump and Stokes powers are calculated corresponding to input pump power variation. To achieve this aim, we have considered the mentioned typical SMF with the variation of the input pump power upto140 mW using a step size of 10 mW power for the 30 km long fiber with 500 m length division.



(a)



(b)

Fig. 6 Transmitted pump (a) and Stokes power (b) vs. input pump power for different fiber lengths.

Fig. 6(a) shows that the transmitted pump power is depleted after a certain incident pump power for any certain fiber length. For shorter length of fiber as shown in Fig. 6(a) (in this case for less than 3 km), the SBS threshold requirements cannot be fulfilled and pump power cannot be transferred to Stokes wave Stokes even though the incident pump power is set at the maximum value of 140 mW. It can be also observed in Fig. 6(b) that the Stokes power increases with input pump power for every different length of fiber up to saturation power. The power transferring from incident signal to Stokes wave can be clearly distinguished in Fig. 6(a, b) such that the transmitted pump power becomes depleted according to Stokes saturation occurrence.

For the threshold power definition regarding various criterion, the earlier results on backscattered Stokes and transmitted power can be adapted in a versatile form of the conversion efficiency percentage, $(I_S(0)/I_P(0) \times 100)$ which is defined in terms of input pump power variation. Fig. 7 indicates the conversion efficiency percentage versus both input pump power and the PCF length which begins to grow sharply at around 1% against input pump power. Based on Smith's definition [13] that considered the input power for which the backscattered Stokes equals incident power at z=0, the threshold power is introduced as Eq. 5. There are two reasons that this definition doesn't match practically, firstly this equation is derived without considering the depletion speculation. Second, the definition condition of threshold is not practically achievable, because the Stokes power is lower than the input pump power due to the pump depletion. Then, the threshold power is defined as an input pump power which is able to provide 1% conversion efficiency at each fiber length.

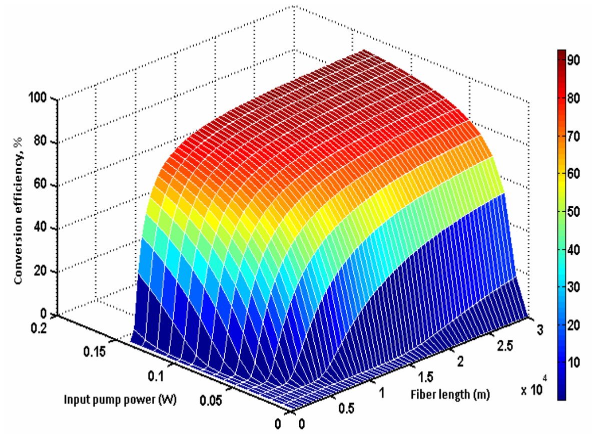


Fig. 7 Conversion efficiency vs. input pump power for different lengths of fiber.

According to the threshold power definition based on 1% criterion, threshold condition is achievable by relating the two results of pump and Stokes power for a specific effective area and Brillouin gain of SMF as shown in Fig. 8. The most typical and common SMFs with the value of 50 and 80 μm^2 of effective area and

1.091×10^{-11} and 5×10^{-11} m/W Brillouin gain have been studied as typical examples to explore the threshold power dependences. Then, the threshold power is defined as an input pump power which is able to provide 1% criterion for every special length with specific A_{eff} and g_B.

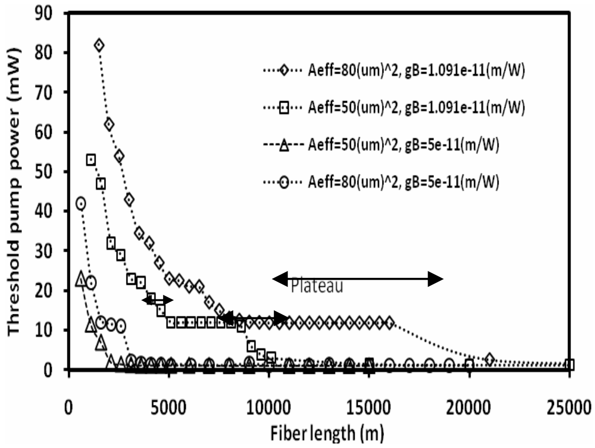
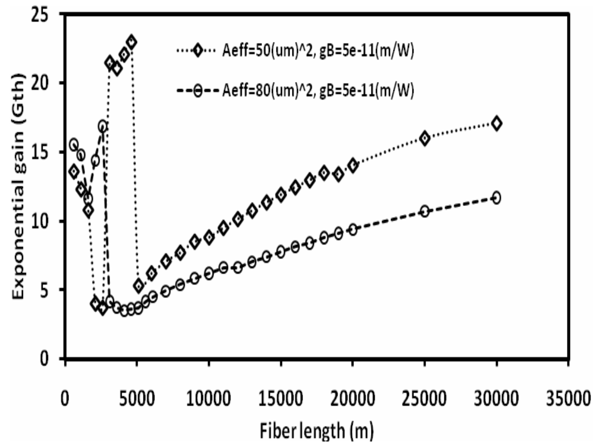


Fig. 8 Threshold pump power vs. fiber length, corresponding to different values of A_{eff} and g_B using 1% criterion.

As shown in Fig.8 this analysis shows that, the threshold pump power depends strongly and explicitly on the length, effective area, and Brillouin gain. The same threshold power trend can be observed which sharply decreases by increment of length up to a special length providing a plateau. Noting that there has to be a tradeoff between the A_{eff} and g_B contents, perhaps it can bring about another decrease after the plateau. This trend shows that after a special length, the threshold power doesn't depend on the fiber length anymore. The value of this length is evaluated by SBS characterizing parameters as well as A_{eff}, g_B and fiber loss.

To achieve the last aim of these calculations for practical usage of the most common Eq. (4) for practical recognizing the threshold power, we calculate the exponential gain, $G_{th} = g_B P_{th}\left(L_{eff}/A_{eff}\right)$. Foremost, G_{th} is commonly taken to be a constant of nearly 21, independent of any effective SBS characterizing parameters. However with this simulation, G_{th} can be calculated by the precisely evaluated threshold power, according to the obtained values shown in Fig. 8. To consider the most general conditions, the G_{th} dependences on three main SBS characterizing parameters are classified and depicted in Fig. 9 (a, b).



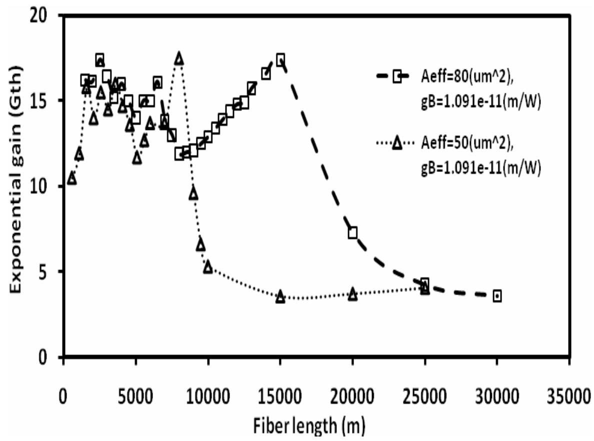


Fig. 9 G_{th} vs. fiber length corresponding to different kinds of A_{eff} and g_B.

As the threshold power depends on these parameters as discussed before, *Gth* also treats in the same behavior. Therefore, it is evident that G_{th} is not a constant as commonly taken in the literature. The higher values of g_B and smaller values of effective area provide the threshold power requirement with the shorter long fiber as shown in Fig. 9 (a, b). For $g_B = 1.091\times10^{-11}$ mW with different values of A_{eff}=50 and 80 μm^2, G_{th} is fluctuating from range of 10 to a special length of providing the maximum value of 18 and then it makes a sharp drop stabilizing gradually by a value of between 4 to 6. Therefore, the limiting value of G_{th} becomes 4 for the longer length and between 10 and 18 is for relatively shorter length. These theoretical predictions matches well the minimum calculated value of G_{th} in Ref. [7] as a limiting level of 4~6. In the case of $g_B = 5\times10^{-11}$ mW, which can provide SBS threshold with a shorter required length, G_{th} trend is different in the last step of the trend stabilizing and it is increasing gradually. Since P_{th} is a constant value of around 1mW as shown in Fig. 8 and according to this relation of $G_{th} = g_B P_{th}\left(L_{eff}/A_{eff}\right)$, the fraction of $\left(L_{eff}/A_{eff}\right)$ is playing the same role as the former case for different value of A_{eff}, this different step is attributed to the higher value of g_B.

VI. CONCLUSION

Briefly, we have studied an approach to calculate SBS threshold power provided by most common types of single mode fibers as a Brillouin gain medium. Threshold pump power corresponding 1% criterion has been evaluated versus fiber length and incident pump power by considering pump depletion and fiber linear loss which indicates a good agreement with the obtained experimental and reported simulation results. The Brillouin threshold gain was also calculated precisely for the most common various available single mode fibers which is varying from 4 to 23 depending on the used conditions.

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