Numerical Calculation of Coupling Efficiency for an Elegant Hermite-Cosh-Gaussian Beams

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ABSTRACT—The ABCD matrix method is used to simplifying the theoretical coupling efficiency calculation of Elegant Hermite-Cosh-Gaussian (EHChG) laser beams to a Single Mode Fiber (SMF) with a quadric lens formed on the tip. The integrals of coupling efficiency relation are calculated numerically by Boole method. Meanwhile, the structure parameters of surface-lensed fiber are optimized in numerical simulation to achieve maximum coupling efficiency. Results can give some guidance suggestions for designing suitable micro lenses in order to coupling the EHChG laser beams to the SMF.

KEYWORDS: ABCD matrix, Coupling efficiency, Elegant Hermite-Cosh-Gaussian beams, Fiber micro-lenses, Single-mode fiber.

I. INTRODUCTION

Introducing an effective technique for coupling between a laser diode (LD) and an optical fiber is one of the most basic concerns in optical communication systems such as pumping erbium-doped optical fiber amplifier (EDFA) [1]-[3]. Two important techniques for coupling between LDs and SMFs are intrinsic micro-lens techniques and external bulk lens techniques [3]. These techniques form lens-like structure directly on the fiber end to improve mechanical stability, simplicity of fabrication and ease of packaging [4].

In order to simplify the theoretical coupling efficiency calculation, authors presented the special ABCD matrix of the parabolic lens [5]-[10]. They used it for calculating the coupling efficiency of fundamental TEM_{00} laser mode to a SMF. However, they do not extend this concept for other types of laser mode such as Elegant Hermit-Gaussian (EHG). The EHG beams have been presented and studied by many authors and compared with the standard Hermit-Gaussian beams in rectangular symmetry [11]-[14]. Likewise, Cosh-Gaussian (ChG) beams have been also studied [15], [16]. The propagation properties of an Elegant Hermit-Cosh-Gaussian (EHChG) beam through a first-order ABCD system as a more generalized case of EHG and ChG beams are derived by Song Yu *et al.* in 2002 [17].

In this paper, we will express the coupling efficiency of some different EHChG beams to a SMF. The paper is organized as follows. In Section 2.1, we study the propagation properties of EHChG laser beam through an ABCD optical system. In Section 2.2, we employ the ABCD transfer matrix for quadric surface lens under paraxial approximation in order to simplify the coupling efficiency calculation of the EHChG laser beam to a SMF. In Section 2.3, we calculate the cumbersome integrals of the coupling efficiency relation numerically by Boole method and optimize it in numerical simulation. In Section 3, we compare our results with the recently published analytical results in [10]. However, it is done only for TEM_{00} laser mode. Finally, the conclusions are given.

II. THEORY

The basic coupling scheme is shown in Fig. 1. The quadric interface micro-lens forming on the fiber tip between parts 1 and 2 is designed to match the modes of the EHChG laser beam and the SMF. We suppose that the optical axis Z coincides with the axis of SMF optics.

Therefore, x-y surface is parallel to the optical fiber end. n_0 and n_0' show the refractive indices of the gap material and the lens, respectively. LD is placed at a distance *L* from the lens surface, and *d* is the thickness of the lens.

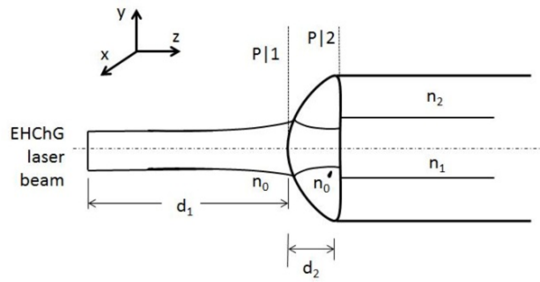

Fig. 1 Schematic diagram of coupling an EHChG laser beam to a single-mode fiber via a quadric lens.

A. *Propagation Properties of EHChG Laser Beam through an ABCD System*

In the *x* direction, the field distribution of EHChG Beam at the waist plane is:

$$\psi(x,0)=\psi_0 H_m\left(\frac{x}{w_0}\right)\cosh(\Omega_0)\exp\left(-\frac{x^2}{w_0^2}\right) \quad (1)$$

where, *m* is the mode index of Hermite polynomial, w_0 is the waist width of the Gaussian amplitude distribution, ψ_0 is the amplitude at the central position of $x=z=0$ and Ω_0 is the cosh function parameter. By using the normalized coordinate $x'=x/w_0$ and decentered parameter $b=w_0\Omega_0$ (suppose $b>0$), we have:

$$\psi(x_0,0)=\psi_0 H_m(x')\cosh(bx')\exp(-x'^2) \quad (2)$$

Equation (2) demonstrates the electric field distributions of EHChG laser beams at the initial plane ($z=0$) which is shown in Fig. 2 for various decentered parameters $b=1,2,3$ and the mode indexes $m=0,1,2$. The EHChG beams with the indexes $m=b=0$ describe the simple Gaussian beam, which is a good approximation of the mode of the SMF.

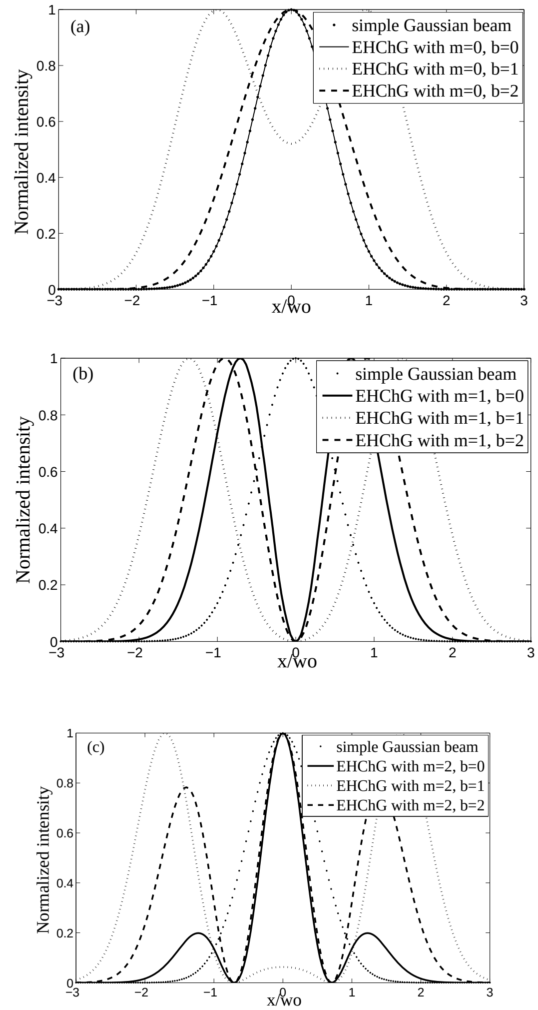

Fig. **2** Electric field distributions of EHChG laser beams at the initial plane, normalized to unity at $x'=0$ for various decentered parameters $b=1,2,3$. The mode indexes $m=0,1,2$ are shown in (a), (b) and (c), respectively.

Song Yu *et al.* [17] calculated the propagation of EHChG beams through a first-order optical system by an ABCD transfer matrix via employing the generalized Huygens-Fresnel diffraction integral [18] in the form:

$$\psi(x',z)=\frac{\psi_0}{2}A^{\frac{m}{2}}\left(A+\frac{B}{q_0}\right)^{-\frac{m+1}{2}}\exp\left(-\frac{n_2 q_0 x'^2}{n_1 q}\right)\times \exp\left[\frac{b^2}{4\left(1+\frac{A}{B}q_0\right)}\right](I_+ + I_-) \quad (3)$$

where

$$I_{\pm} = \exp\left(\pm\frac{bq_0}{B+Aq_0}x'\right)H_m\left[\frac{2q_0x' \pm bB}{2\sqrt{Aq_0\left(B+q_0A\right)}}\right] \tag{4}$$

where n_1 and n_2 are refractive indices of the first and second medium, respectively. q is the complex parameter related to the curvature radius R and beam width w introduced as:

$$\frac{1}{q} = \frac{1}{R} - i\frac{\lambda}{\pi w^2} \tag{5}$$

Suppose q_0 is the complex parameter at the waist plane of $z=0$, then according to ABCD law, q and q_0 are connected by:

$$\frac{1}{q} = \frac{C+D/q_0}{A+B/q_0} \tag{6}$$

Thus for three dimensions coordinate, it is acceptable to write:

$$\psi(x',y',z) = \psi(x',z)\times\psi(y',z) \tag{7}$$

where $\psi(y',z)$ is defined as $\psi(x',z)$ in Eq. (3).

B. *Establishment of Transfer Matrices Model of a Quadric Interface-Lensed Fiber*

A unified expression for ABCD transfer matrix of different quadric surface micro-lens under paraxial approximation is introduced by J. Huang and H. Yang as [10]:

$$\begin{pmatrix} 1 & 0 \\ \dfrac{1-n}{nP} & \dfrac{1}{n} \end{pmatrix} \tag{8}$$

The different amounts of P in Eq. (8) describe different structures of quadric surface micro-lens such as elliptical, hyperbolic, spherical and parabolic lenses, which are introduced in Table 1. So for the scheme that is shown in Fig. 1 we can write an ABCD matrix as:

$$M = \begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ (1-n)/(np) & 1/n \end{pmatrix}\begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \tag{9}$$

Table **1** Different structures of quadric surface micro-lens

Kinds of surface	Mathematical relation	Amounts of P
Elliptical and hyperbolic	$\frac{x^2}{a^2} \pm \frac{y^2}{b^2} = 1$	$P = b^2/a$
Spherical	$x^2 + y^2 = r^2$	$P = r$
Parabolic	$x^2 = 2tz$	$P = t$

C. *Theory of Coupling Efficiency Calculation*

The mode field of the source LD is described by an EHChG beam, meanwhile the mode field of SMF can be approximated by a Gaussian field distribution. For the weakly guiding step-index optical fibers, the waist spot size w_f, is defined as the radial distance at which the field amplitude is e^{-1} of its maximum that can be approximated by

$$w_f / a = .65 + 1.619V^{-3/2} + 2.879V^{-6} \tag{10}$$

where a is the core radius. The term V is defined by

$$V = \left(\frac{2\pi a}{\lambda_0}\right)\sqrt{n_1^2 - n_2^2} \tag{11}$$

Here λ_0 is the free-space wavelength and n_1 and n_2 are the core and cladding refractive indices, respectively.

The fundamental mode field ψ_f of the fiber is assumed to be well approximated by a Gaussian beam [7].

$$\psi_f = \exp\left[-\left(\frac{x^2+y^2}{w_f^2}\right)\right] \tag{12}$$

The field ψ_u on the plane 1 at the output of the LD at distance L from the lens surface is described by Eq. (2). Similarly, the lens-transformed laser field on the fiber plane 2 can be expressed by Eq. (7) with ABCD matrix of Eq. (9).

According to the well-known overlap integral, the LD to fiber coupling efficiency via a quadric interface lens on the fiber tip is obtained as [5]-[8]:

$$\eta = \frac{\left|\iint \psi_v^* \psi_f dxdy\right|}{\iint |\psi_v|^2 dxdy \iint |\psi_f|^2 dxdy} \qquad (13)$$

In order to overcome the cumbersome integrals of Eq. (15), we employ the numerical Boole method [19].

As in [10], if the LD output is approximated by simple elliptical Gaussian beam, the coupling efficiency formula can be achieved by simple relation as:

$$\eta = \frac{4w_{2x}w_{2y}}{\left[\left(w_f^2+w_{2x}^2\right)^2+\frac{k_2^2 w_f^4 w_{2x}^4}{4}\left(\frac{1}{R_{2x}}-\frac{1}{R_f}\right)^2\right]^{1/2}} \times \frac{w_f^2}{\left[\left(w_f^2+w_{2x}^2\right)^2+\frac{k_2^2 w_f^4 w_{2y}^4}{4}\left(\frac{1}{R_{2y}}-\frac{1}{R_f}\right)^2\right]^{1/2}} \qquad (14)$$

where $k_2 = 2\pi n_0/\lambda_0$ is the wave number in the quadric lens medium, w_{2x} and w_{2y} are the lens-transformed spot sizes and R_{2x} and R_{2y} represent radius of curvature of refracted wave front in the *x* and *y* directions.

III. NUMERICAL RESULTS

In order to couple the EHChG laser beams to a fiber, we use quadric surface micro-lens on the fiber tip. The simplified part of this method owes to ABCD method that describes different kinds of quadric surface micro-lens such as elliptical, hyperbolic and parabolic lenses under paraxial approximation. As the electric field relation of EHChG beam is too abstruse, the coupling efficiency relation (13) is not integrable analytically. Therefore, we employ Boole method for numerical integration.

As similar to experimental work [1], we choose the LD's wavelength $\lambda_0 = 1.3\mu m$, and its $w_{1x} = 1.081\mu m$, $w_{1y} = 1.161\mu m$, the SMF mode field radius $w_f = 4.749$, gap material's refractive index $n_0 = 1$, while the refractive index of lens is $n_0' = 1.55$ and the maximum depth of micro-lens is $d = 6\mu m$.

EHChG laser beam for $m = 0$ and $b = 0$ describes the simple elliptical Gaussian beam. Therefore, it is possible to compare numerical simulation of Eq. (13) with accurate result of Eq. (14) which is shown in Fig. 3. This figure illustrates that numerical results are matched with the analytical solution.

To achieve the maximum coupling efficiency, optimization of focal length and structure parameter must be discussed. Fig. (4-6) describe the coupling efficiency of some modes of EHChG laser beam to SMF versus focal length *L* and structure parameter *P*.

It is clear from Fig. 4 that the maximum coupling efficiency for coupling an EHChG laser beam of mode $m = 0$ and $b = 0$ is 99.988% while $P = 7.094$ and $L = 12.187$. Also from Fig. 5, one can find that the maximum coupling efficiency for coupling an EHChG laser beam of mode $m = 0$ and $b = 2$ is 91.65% while $P = 15.62\mu m$ and $L = 20.83\mu$m. Likewise, Fig. 6 shows that the maximum coupling efficiency for coupling an EHChG laser beam of mode $m = 2$ and $b = 0$ is 46.94% while $P = 3.07\mu m$ and $L = 5.30\mu$m.

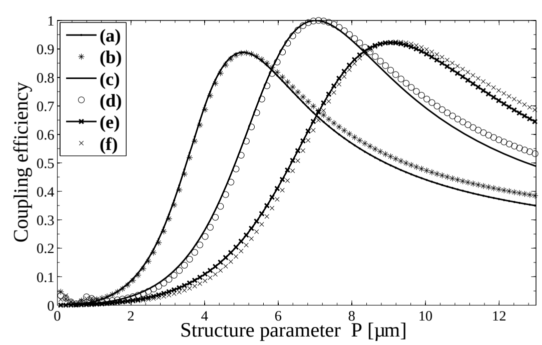

Fig. **3** Comparison of numerical and analytical results. (a) Analytical result for L=8.19 μm. (b) Numerical result for L=8.19 μm. (c) Analytical result for L=12.1875 μm. (d) Numerical result for L=12.1875 μm. (e) analytical result for L=16.19 μm. (f) numerical result for L=16.19 μm.

To sum up, the maximum coupling efficiencies of some different EHChG modes for corresponding structure parameter P under certain focal length L demonstrate in table 2. Meanwhile, these maximum values are shown in Fig. 7.

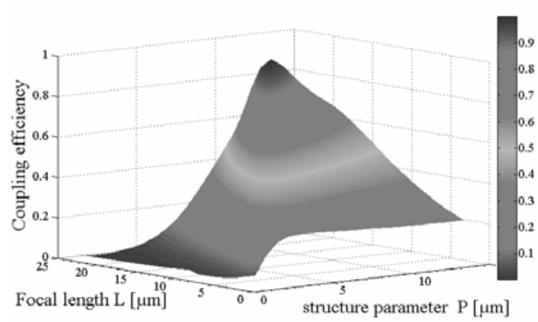

Fig. **4** Coupling efficiency for EHChG laser beam of $m=0$ and $b=0$ versus structure (P) and focal length (L).

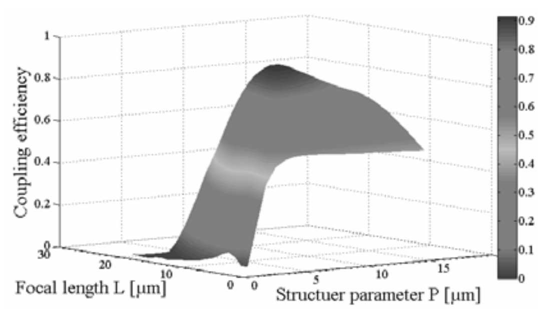

Fig. **5** Coupling efficiency for EHChG laser beam of $m=0$ and $b=2$ versus structure (*P*) and focal length (*L*)

So, with respect to the beam profile, the maximum coupling efficiency is different. In other words, the coupling efficiency improves whenever the beam shape is similar to the fiber mode.

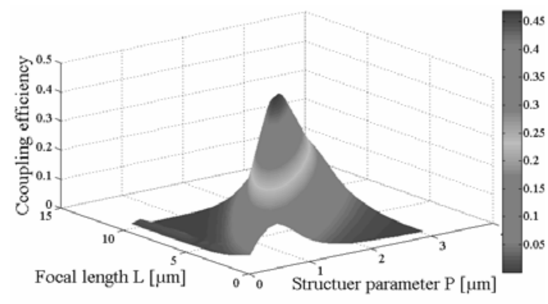

Fig. **6** Coupling efficiency for EHChG laser beam of $m=2$ and $b=0$ versus structure (P) and focal length (L)

Figure 4 shows that the coupling efficiency for coupling an EHChG laser beam of the modes $m=0$ and $b=0$ reach near 100% for $P=7.094$ and $L=12.187$. Whereas, in the case $m=0$, by increasing the mode index *b*, the beam shape varies from simple Gaussian beam (see Fig. 2a). Therefore, the maximum coupling efficiency decreases, as shown in Fig. 7. Besides, the beams correspond to the odd m that have minimum in center (see fig. 2b) are rarely coupled to the fiber. Fig. 7 shows that in the case $m=2$, the maximum coupling efficiency decreases by increasing *b* up to 1.2. Because the center peak of the beam decreases and the satellite peak increases (see Fig. 2c). Continuously, by increasing the b from 1.2, the center peak of the beam increases. So, the maximum coupling efficiencies increase too.

Table **2** Summary of maximum coupling efficiency of some different modes for corresponding structure parameter (P) under certain focal length (L)

EHChG Mode index (m)	EHChG Mode index (b)	Maximum coupling efficiency (%)	P (μm)	L (μm)
0	0	99.988	7.09375	12.1875
0	1	99.894	9.01562	14.9218
0	2	91.647	15.6250	20.8281
0	3	67.197	30.8125	25.6093
2	0	46.941	30.7031	5.29687
2	1	11.377	29.6875	4.23437
2	2	30.718	3000.00	60.0000
2	3	35.805	1023.00	53.3000

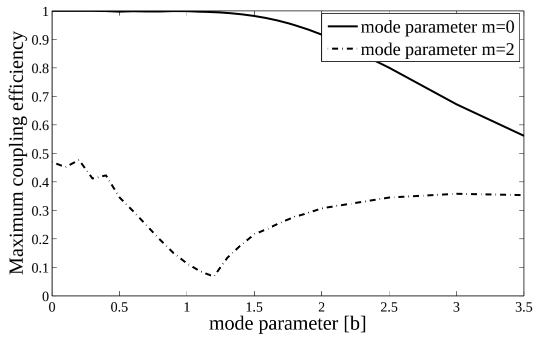

Fig. **7** Maximum coupling efficiency for EHChG laser beam of $m = 0, 2$ versus mode parameter *b*.

IV. Conclusion

The quadric surface micro-lens on the fiber tip was employed in order to achieve the maximum coupling efficiency for coupling the EHChG laser beams to a SMF. A unified ABCD matrix was used for different kinds of quadric surface such as elliptical, hyperbolic and parabolic lenses to simplify the theoretical coupling efficiency calculation. In order to achieve the maximum coupling efficiency, the structure parameters of surface-lensed fiber were optimized by numerical simulation. The advantage of this study is simplifying the calculation.

Results show that the coupling efficiency improves whenever the beam shape is similar to the fiber mode. For the EHChG laser beam of mode $m = 0$, the maximum coupling efficiency decrease from near 100% by increasing the mode index b. On the other hand, the beams correspond to odd mode parameters m are rarely coupled to the fiber. For mode parameter m=2 by increasing b up to 1.2 the maximum coupling efficiencies decrease and then increases as shown in Fig. 7. This technique will be useful to fabricate the optimum quadric interface micro-lens for coupling an EHChG laser beams to a SMF.

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