Simulation of an Airy Beam with Optical Vortex under Fractional Fourier Transforms

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Abstract— First, this study obtained the fields of an Airy beam (AiB) with optical vortex (OV) for a Fourier transform (FT) system and a fractional Fourier transform (fractional FT) system; thereafter, their intensity and phase patterns were simulated numerically. The splitting on each line of the phase pattern indicates the position of an OV. The results show that the OV position will change when the power of the fractional FT (*p*) changes. Moreover, the uniformity of the spot beam disappears for the beam with OV. Further, the characteristics of an AiB such as number, width, height, uniformity of the spot beam and the effective beam size will change when there is a change in the values of *p* and *z*.

KEYWORDS: Fractional FT, optical vortex, Airy beam.

I. INTRODUCTION

AiB has numerous features and they were considered in the last decade. Although the concept of the Airy wave package dates back 30 years [1], the counterpart in optics was only demonstrated recently. In 2007, an AiB was produced experimentally [2]. In 2008, vector evolution, angular momentum and AiB phase behavior were investigated [3, 4]. Several methods, including a method of the Wigner distribution function for describing the features of an AiB have been presented [5]. Many investigations have been made in this regard, such as the propagation of the AiB in free space, in the nonlinear medium [6], in the turbulence [7] and of the misaligned optical system [8]. The propagation of an AiB with OV [9, 10] and propagation of an Airy-Gaussian beam with OV in the chiral medium are investigated [11]. In 2012, the researchers by studying the AiB showed that the scope of the analysis of the AiB characteristic for its propagation through the fractional FT system is more than the FT system. In this case, assuming that a piece of graded-index fiber with proper length L is required for performing a FT of an input image. If the graded-index fiber is cut into pieces, a piece of length *pl* (*p*<1) just performs the fractional FT of the input image [12]. The research in the AiB with OV at the fractional FT has not been reported. Previously, a study was conducted on the AiB that was propagated through the fractional FT system. Now, we are inspired by the work of the researchers [12], studying the AiB with OV that propagates through the fractional FT system and then simulate its intensity and phase pattern numerically. We want to know, how the AiB with OV behaves by changing the order of the fractional FT and distance in the optical system.

II. THEORETICAL CONCEPT

A. *FT System*

The field of an AiB is defined with OV as [9]:

$$E(x,y,z=0)=Ai(x/x_0)\exp(ax/x_0)Ai(y/y_0)\times \exp(ay/y_0)\left((x-x_d)+i(y-y_d)\right)^l . \quad (1)$$

where the parameters x_0 and y_0 are transverse scales, and *a* is the truncation factor exponential; which characterizes beams, *l* is the topological charge of the vortex. For simplicity, we choose the unit topological charge and the OV core center is in the coordinates (x_d, y_d). The Fresnel integral can be used to calculate the field of the beam at distance z in the FT system [13]. On substituting Eq. (1) into (2), the field of the AiB with OV can be obtained, after it propagates a distance z.

$$
\begin{aligned}
&E(x,y,z)=\frac{ik}{2\pi z}\times \\
&\iint dx_1 dy_1 \exp\left\{\frac{ik}{2z}\left[(x_1-x)^2+(y_1-y)^2\right]\right\}\times \\
&E(x_1,y_1,z=0)=\frac{ik}{2\pi z}\iint dx_1 dy_1 \exp\left\{\frac{ik}{2z}\left[(x_1-x)^2+\right.\right. \\
&\left.\left.(y_1-y)^2\right]\right\} Ai\left(\frac{x_1}{x_0}\right)Ai\left(\frac{y_1}{y_0}\right)\exp\left(\frac{ax_1}{x_0}+\frac{ay_1}{y_0}\right)\times \\
&\left[(x_1-x_d)+i(y_1-y_d)\right]
\end{aligned}
\tag{2}
$$

and

$$
\begin{aligned}
&E(x,y,z)=\frac{ik}{2\pi z}\exp\left(\frac{ax}{x_0}-\frac{ia^2z}{2kx_0^2}+\frac{ay}{y_0}-\frac{ia^2z}{2ky_0^2}\right)\times \\
&\left\{\int dx_1 \exp\left(\frac{ik}{2z}\left[\left(x_1-\left(x-\frac{iaz}{x_0k}\right)\right)^2\right]\right)Ai\left(\frac{x_1}{x_0}\right)(x_1-x_d)\right. \\
&\times\int dy_1 \exp\left(\frac{ik}{2z}\left[y_1-\left(y-\frac{iaz}{y_0k}\right)\right]^2\right)Ai\left(\frac{y_1}{y_0}\right)+ \\
&\int dx_1 \exp\left(\frac{ik}{2z}\left(x_1-\left(x-\frac{iaz}{x_0k}\right)\right)^2\right)Ai\left(\frac{x_1}{x_0}\right)\times \\
&\left.\int dy_1 \exp\left(\frac{ik}{2z}\left[y_1-\left(y-\frac{iaz}{y_0k}\right)\right]^2\right)Ai\left(\frac{y_1}{y_0}\right)(y_1-y_d)\right\}
\end{aligned}
\tag{3}
$$

By utilizing the convolution theorem, we can write the field of the beam as follows [12].

$$
\begin{aligned}
&E(x,y,z)=\frac{ik}{2\pi z}\frac{x_0}{\sqrt{2\pi}}\frac{y_0}{\sqrt{2\pi}}\left(\sqrt{\frac{-iz}{k}}\right)^2\exp\left(\frac{ax}{x_0}\right)\times \\
&\exp\left(\frac{ay}{y_0}\right)\left\{\left[\int-\frac{z\xi}{k}\exp\left(\frac{-ix_0^3\xi^3}{3}-\frac{iz\xi^2}{2k}-i\xi x+\frac{az\xi}{x_0k}\right)d\xi\right.\right. \\
&\left.\int\left(x-\frac{iaz}{x_0k}-x_d\right)\exp\left(\frac{-ix_0^3\xi^3}{3}-\frac{iz\xi^2}{2k}-i\xi x+\frac{az\xi}{x_0k}\right)d\xi\right]\times \\
&\int\exp\left(\frac{-iy_0^3\xi^3}{3}-\frac{iz\xi^2}{2k}-i\xi y+\frac{az\xi}{y_0k}\right)d\xi+ \\
&i\int\exp\left(\frac{-ix_0^3\xi^3}{3}-\frac{iz\xi^2}{2k}-i\xi x+\frac{az\xi}{x_0k}\right)d\xi\times \\
&\left[\int\frac{z\xi}{k}\exp\left(\frac{-iy_0^3\xi^3}{3}-\frac{iz\xi^2}{2k}-i\xi y+\frac{az\xi}{y_0k}\right)d\xi+\right. \\
&\left.\left.\int\left(y-\frac{iaz}{y_0k}-y_d\right)\exp\left(\frac{-iy_0^3\xi^3}{3}-\frac{iz\xi^2}{2k}-i\xi y+\frac{az\xi}{y_0k}\right)d\xi\right]\right\}
\end{aligned}
\tag{4}
$$

Equation (5) can be expressed as follows

$$
\begin{aligned}
&E(x,y,z)=\frac{x_0y_0}{(2\pi)^2}\exp\left(\frac{ax}{x_0}+\frac{ay}{y_0}\right)\exp\left(\frac{-az^2}{2k^2x_0^4}-\frac{az^2}{2k^2y_0^4}\right)\times \\
&\exp\left(\frac{izx}{2kx_0^3}+\frac{izy}{2ky_0^3}+\frac{iz^3}{24k^3x_0^3}+\frac{iz^3}{24k^3y_0^3}-\frac{iz^3}{8k^3x_0^6}-\frac{iz^3}{8k^3y_0^6}\right)\times \\
&\left\{\left[\frac{ik}{z}Ai'\left(\frac{x}{x_0}-\frac{z^2}{4k^2x_0^4}+\frac{iaz}{x_0^2k}\right)+\left(x-\frac{iaz}{x_0k}-x_d\right)\times\right.\right. \\
&\left.Ai\left(\frac{x}{x_0}-\frac{z^2}{4k^2x_0^4}+\frac{iaz}{x_0^2k}\right)\right]\times Ai\left(\frac{y}{y_0}-\frac{z^2}{4k^2y_0^4}+\frac{iaz}{y_0^2k}\right)+ \\
&iAi\left(\frac{x}{x_0}-\frac{z^2}{4k^2x_0^4}+\frac{iaz}{x_0^2k}\right)\times\left[\frac{ik}{z}Ai'\left(\frac{y}{y_0}-\frac{z^2}{4k^2y_0^4}+\frac{iaz}{y_0^2k}\right)+\right. \\
&\left.\left.\left(y-\frac{iaz}{y_0k}-y_d\right)Ai\left(\frac{y}{y_0}-\frac{z^2}{4k^2y_0^4}+\frac{iaz}{y_0^2k}\right)\right]\right\}
\end{aligned}
\tag{5}
$$

B. *Fractional FT*

Figure 1 is the general expression of the field of an AiB with OV passing through an optical system. The Lohmann I optical system and the Lohmann II optical system are equivalent, and are described by the following transfer matrix [14]:

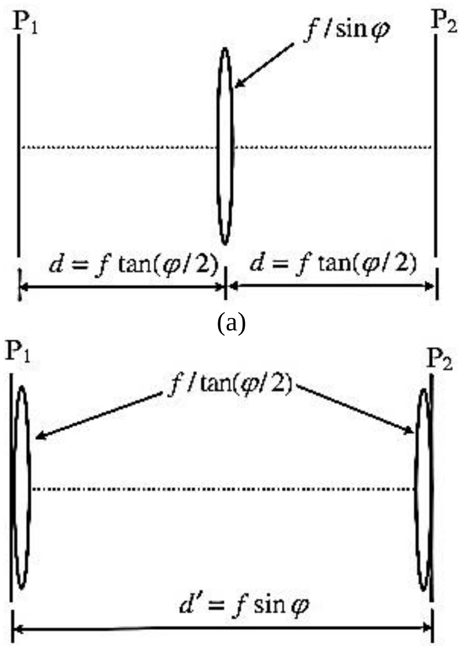


Fig. 1. Optical system for the FrFT, for Lohmann (a) I and (b) systems.

$$R=\begin{pmatrix} A & B \\ C & D \end{pmatrix}=\begin{pmatrix} \cos\varphi & f\ \sin\varphi \\ -1/f\ \sin\varphi & \cos\varphi \end{pmatrix} \qquad (6)$$

where, $\varphi=\pi p/2$ and p is the order of fractional FT [14]. In Fig 1 (a), the distance between the input and output planes is $2d=2f\tan(\varphi/2)$. In Fig 1 (b), the distance between the input and output planes is $d'=f\sin(\varphi)$. Under the paraxial approximation, the Collin formula can be used to calculate the field of the AiB passing through the Lohmann optical system [12].

$$E(x,y,z)=\frac{i}{\lambda B}\exp(ikz) \iint dx_1 dy_1 \left\{\exp\left[\frac{i\pi}{\lambda B}A\left(x_1^2+y_1^2\right)-2\left(xx_1+yy_1\right) D\left(x^2+y^2\right)\right]E\left(x_1,y_1,z=0\right)\right\} \qquad (7)$$

Finally, the field of the AiB with OV under fractional FT after propagating a distance z by employing the convolution theorem, can be formulated as:

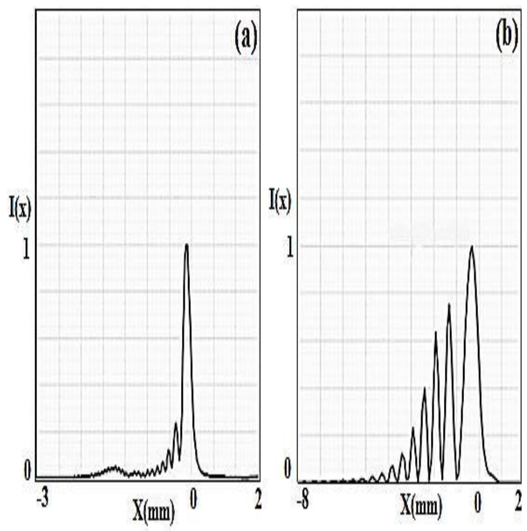
$$E(x,y,z)=\frac{\exp(ikz)}{i\lambda B}\exp\left[\frac{ikD\left(x^2+y^2\right)}{2B}\right] \exp\left(\frac{ax}{x_0}+\frac{ay}{y_0}\right)\exp\left(\frac{-ikx^2}{2BA}-\frac{iky^2}{2BA}+\frac{ia^2B}{2Akx_0^2}\right)\times \left\{\left[\int_{-\infty}^{\infty}d\xi\left(\frac{-B\xi}{kA}+\frac{x}{A}+\frac{iaB}{kAx_0}-x_d\right)\exp\left(-\frac{ix_0^3\xi^3}{3}- \frac{iB\xi^2}{2kA}-i\xi\left(\frac{x}{A}+\frac{iaB}{kAx_0}\right)\right)\int_{-\infty}^{\infty}d\xi\exp\left[\left(-\frac{iy_0^3\xi^3}{3}- \frac{iB\xi^2}{2kA}-i\xi\left(\frac{y}{A}+\frac{iaB}{kAy_0}\right)\right]\right]+\left[i\int_{-\infty}^{\infty}d\xi\exp\left[\left(-\frac{ix_0^3\xi^3}{3}- \frac{iB\xi^2}{2kA}-i\xi\left(\frac{x}{A}+\frac{iaB}{kAx_0}\right)\right]\right]\int_{-\infty}^{\infty}d\xi\left(\frac{-B\xi}{kA}+\frac{y}{A}+\frac{iaB}{kAy_0}- y_d\right)\exp\left[-\frac{iy_0^3\xi^3}{3}-\frac{iB\xi^2}{2kA}-i\xi\left(\frac{y}{A}+\frac{iaB}{kAy_0}\right)\right]\right]\right\} \qquad (8)$$

Eq. (8) can be expressed as follows

$$E(x,y,z)=\frac{\exp(ikz)}{i\lambda B}\exp\left[\frac{ikD\left(x^2+y^2\right)}{2B}\right]\exp\left(\frac{ax}{x_0}+ \frac{ay}{y_0}+\frac{aB^2}{2k^2Ax_0^4}+\frac{aB^2}{2k^2Ay_0^4}\right)\exp\left(\frac{-ikx^2}{2BA}-\frac{iky^2}{2BA}+ \frac{ia^2B}{2Akx_0^2}+\frac{ia^2B}{2Akx_0^2}\right)\exp\left(\frac{iB^3}{24k^3x_0^3A^3}+\frac{iB^3}{24k^3y_0^3A^3}- \frac{iB^3}{8k^3x_0^4A^3}-\frac{iB^3}{8k^3y_0^4A^3}+\frac{ixB}{2kA^2x_0^3}+\frac{iyB}{2kA^2y_0^3}\right)\left\{\left(\left[\frac{ik}{B}\times Ai'\left(\frac{x}{x_0A}-\frac{B^2}{4k^2A^2x_0^2}+\frac{iaB}{kAx_0^2}\right)+\left(\frac{x}{A}-x_d+\frac{iaB}{kAx_0}\right)\times Ai\left(\left(\frac{x}{x_0A}-\frac{B^2}{4k^2A^2x_0^2}+\frac{iaB}{kAx_0^2}\right)\right]\times Ai\left(\frac{y}{y_0A}-\frac{B^2}{4k^2A^2y_0^2}+ \frac{iaB}{kAy_0^2}\right)\right)+iAi\left(\frac{x}{x_0A}-\frac{B^2}{4k^2A^2x_0^2}+\frac{iaB}{kAx_0^2}\right)\times \left[Ai'\left(\frac{y}{y_0A}-\frac{B^2}{4k^2A^2y_0^2}+\frac{iaB}{kAy_0^2}\right)+\left(\frac{y}{A}-y_d+\frac{iaB}{kAy_0}\right)\times Ai\left(\frac{y}{y_0A}-\frac{B^2}{4k^2A^2y_0^2}+\frac{iaB}{kAy_0^2}\right)\right]\right\} \qquad (9)$$

III. Numerical Simulations and Analysis

It was assumed that $\lambda=0.53\mu m$, $f=1000mm$, $x_0=y_0=0.1mm$, $x_d=y_d=0.3mm$ and $a=10^{-5}$. In this simulation, Eq. (5), Eq. (9), paraxial approach and the method of the Wigner distribution function were used. Fig. 2 shows the two-dimensional intensity pattern for AiB. From Fig. 2(a) it can be found that the uniformity of the lateral lob of the beam with OV disappears while, in Fig. 2(b), the peak uniformity of the beam without OV increases. It can also be seen in Fig. 2 (a), that the effective size of the beam is decreased, the main peak is straitened and the width and height of the other peaks are also decreased.



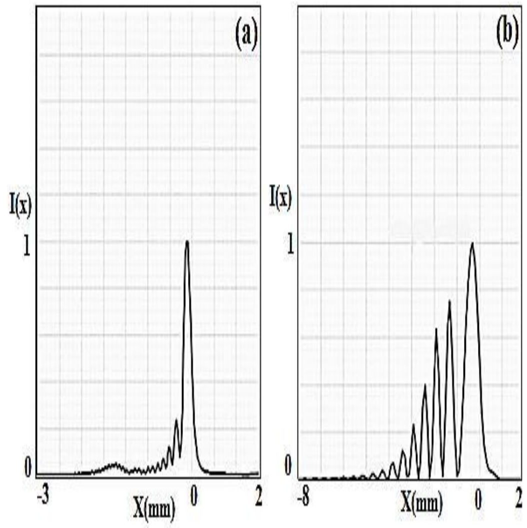


Fig. 2. Intensity normalized in the *x*- direction of AiB; (a) with OV; (b) without OV.

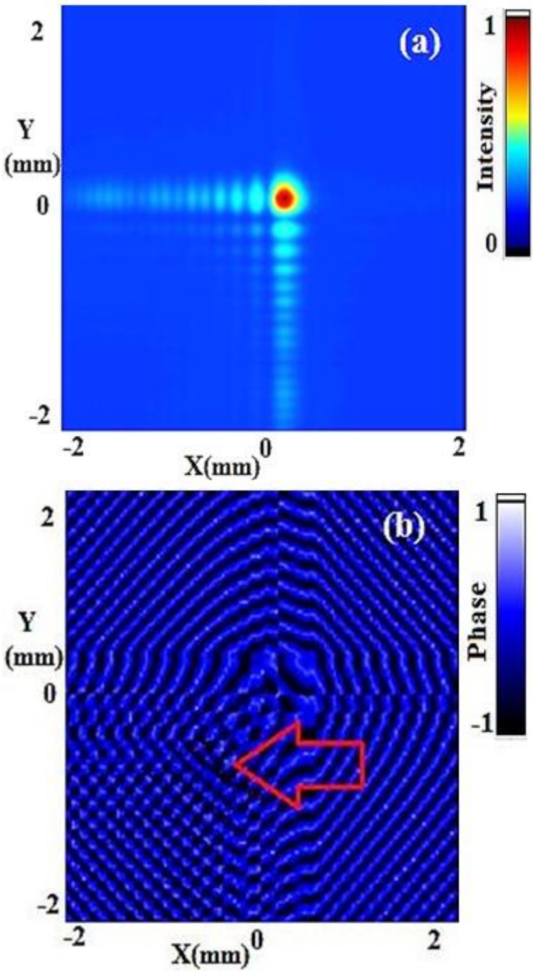
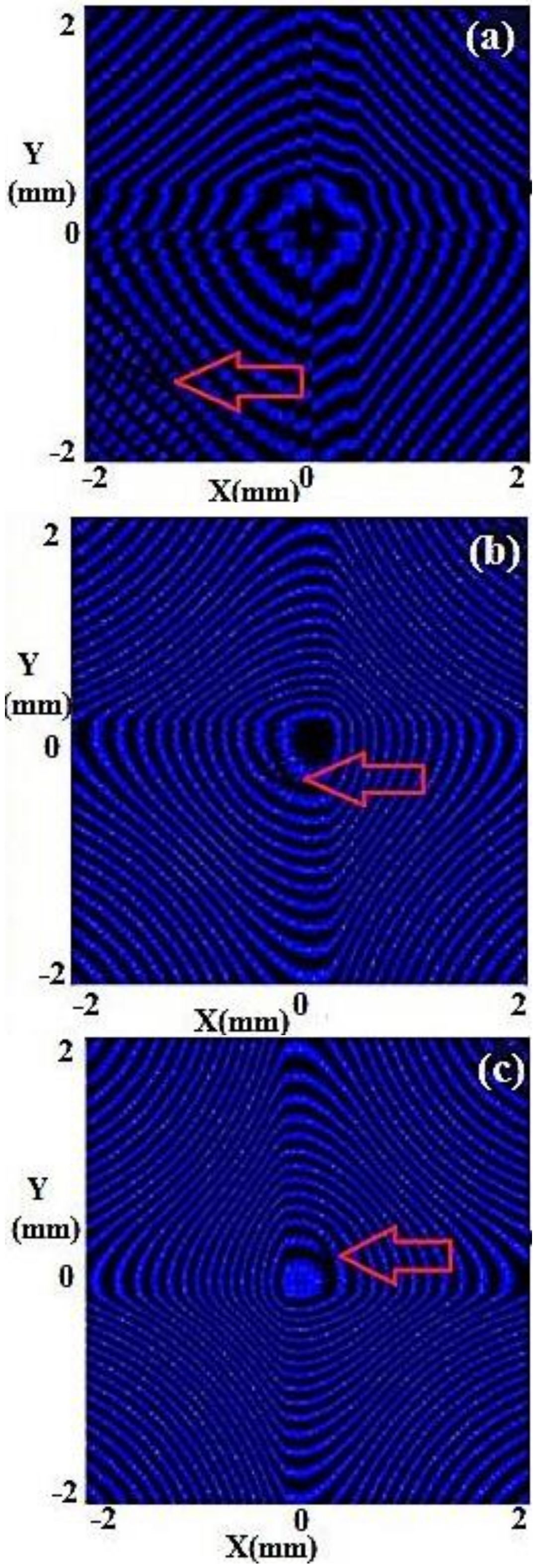


Fig. 3. AiB with OV; (a) intensity and (b) phase pattern.



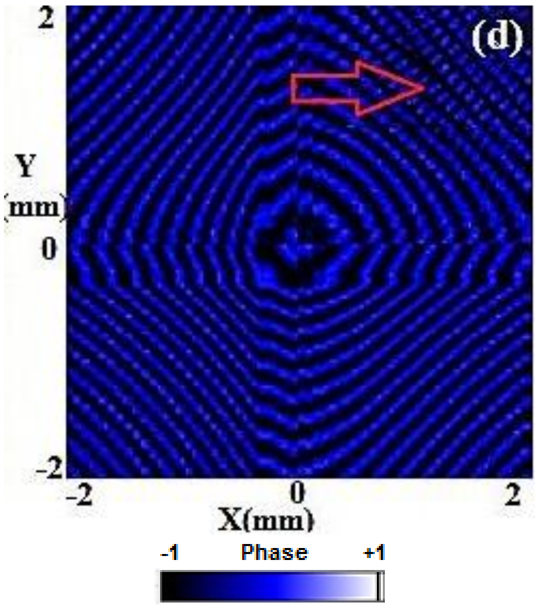
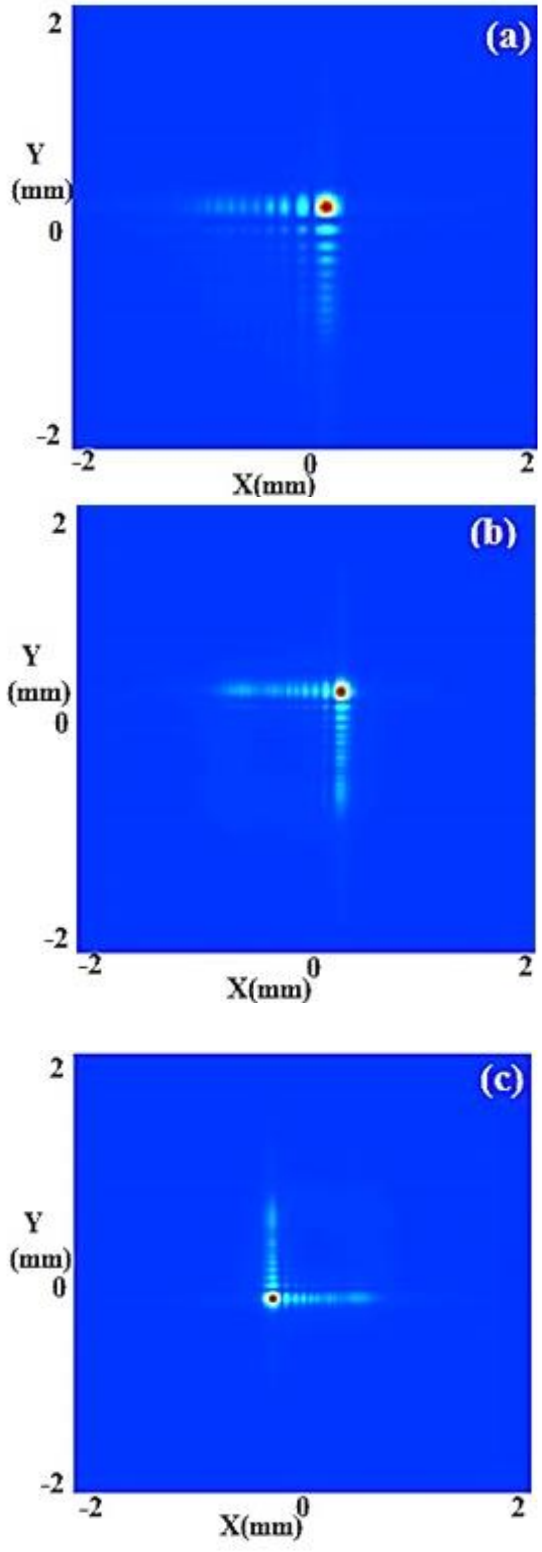


Fig. 4. Phase pattern of the AiB with OV for; the power of fractional FT for (a) p=0.3, (b) p=0.7, (c) p=1.3, (d) p=1.7.

Fig. 3 shows the intensity pattern (a) and phase pattern (b). In Fig. 3 (b), the arrow represents the position of the OV. Fig. 4 shows the phase and intensity patterns of an AiB with OV for the different powers of fractional FT, the direction of the patterns changes, which is a logical conclusion of Eq. (7).

The position of the OV changes with increasing p, and this can be seen in Fig. 4 (a). If the value of p is closer to 1, the effective beam size will decrease. In Fig. 4 (b), with approaching p to 1, the main peak straitens.

In Fig. 5, for p<1, the beam spot of the AiB decreases when the value of p is increased. For 1<p<2, the beam spot of the AiB increases with increasing the value of p. Fig. 6 shows the two-dimensional intensity graphs for different p. In this case, if the value of p is closer to 1 the lateral side lobs will be far from the x axis.



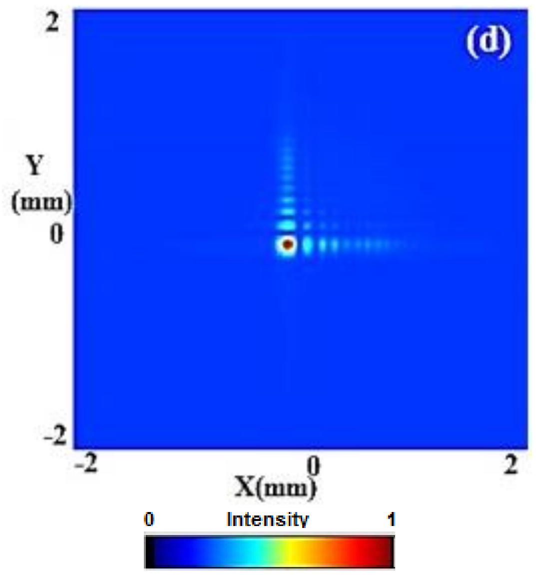
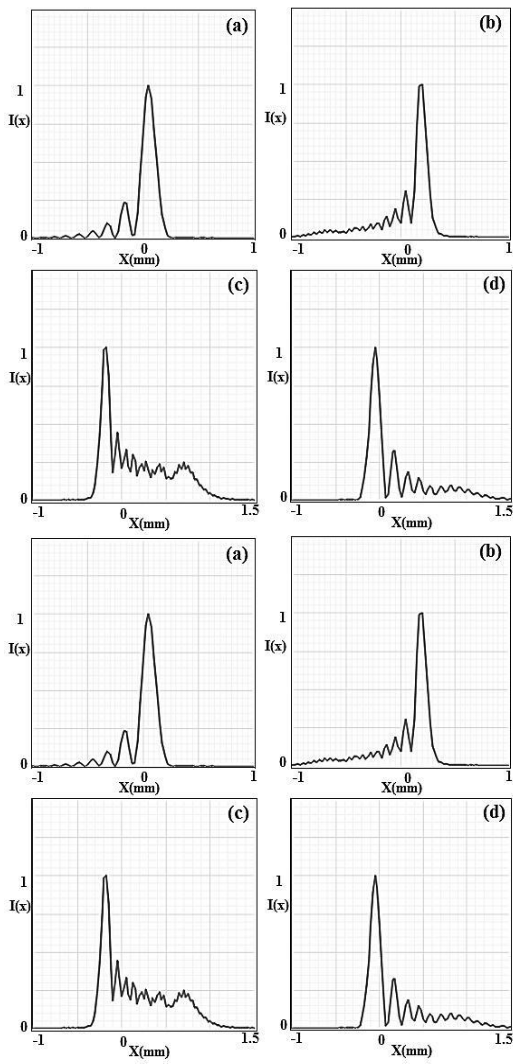


Fig. 5. Intensity pattern of the AiB with OV for; the power of fractional FT for (a) p=0.3, (b) p=0.7, (c) p=1.3, (d) p=1.7.



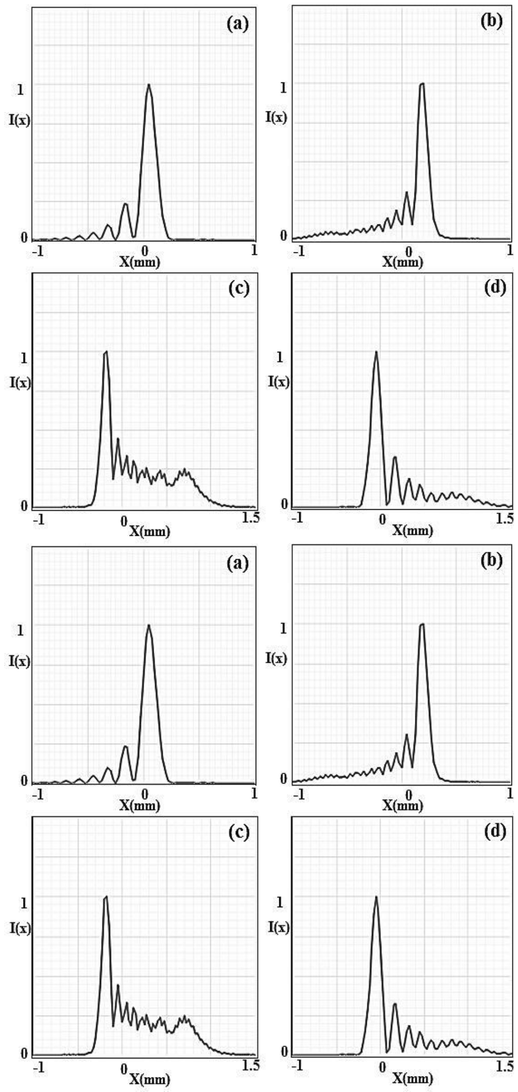


Fig. 6. Intensity normalized in the x - direction of AiB with OV; (a) p=0.3, (b) p=0.7, (c) p=1.3, (d) p=1.7.

Figure 7 shows the two-dimensional spot beam size (S) graph of the AiB with OV for different p. for p<1, the beam spot of the AiB decreases when the value of p is increased. For 1<p<2, the beam spot of the AiB increases with increasing value of p. Figs. 8 and 9 show the phase and intensity patterns for an AiB with OV. We consider the propagation of an AiB with OV and the propagation of an AiB without OV for fixed parameters λ, f, a, x_d, x_0 and by varying the value of z. In this case for each z, there is a unique transfer matrix, which depends on the distance between the two primary and final pages.

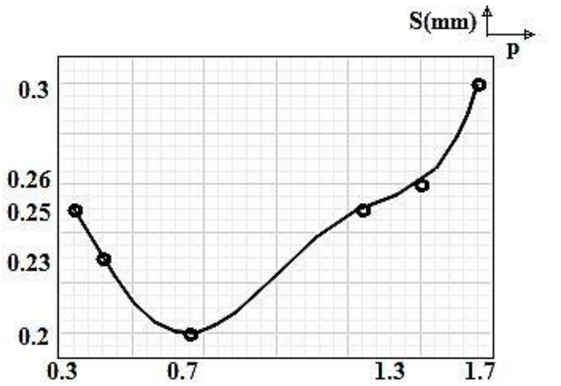
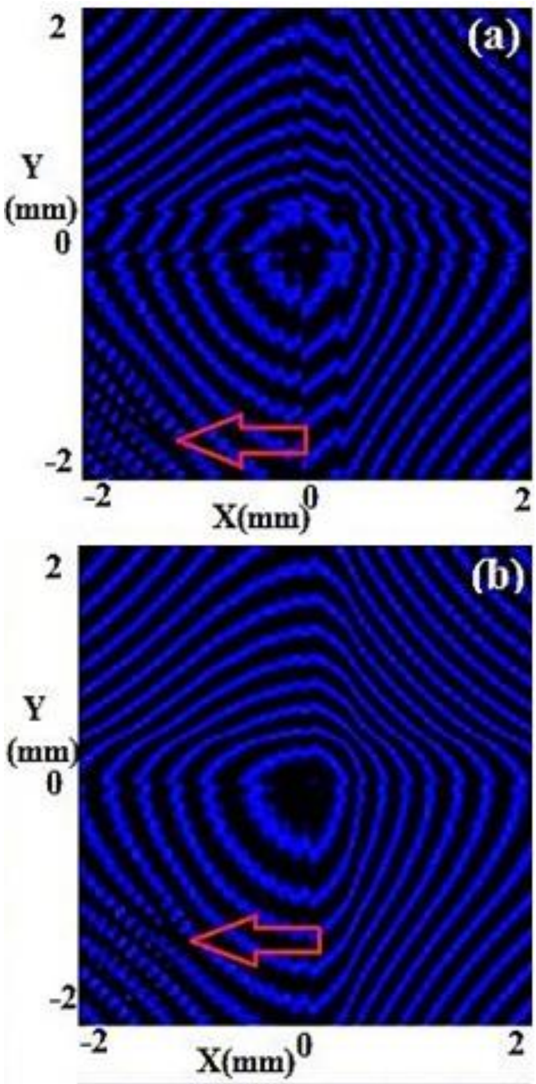


Fig. 7. Spot beam size (S) of the AiB with OV for the powers of the fractional FT (p).



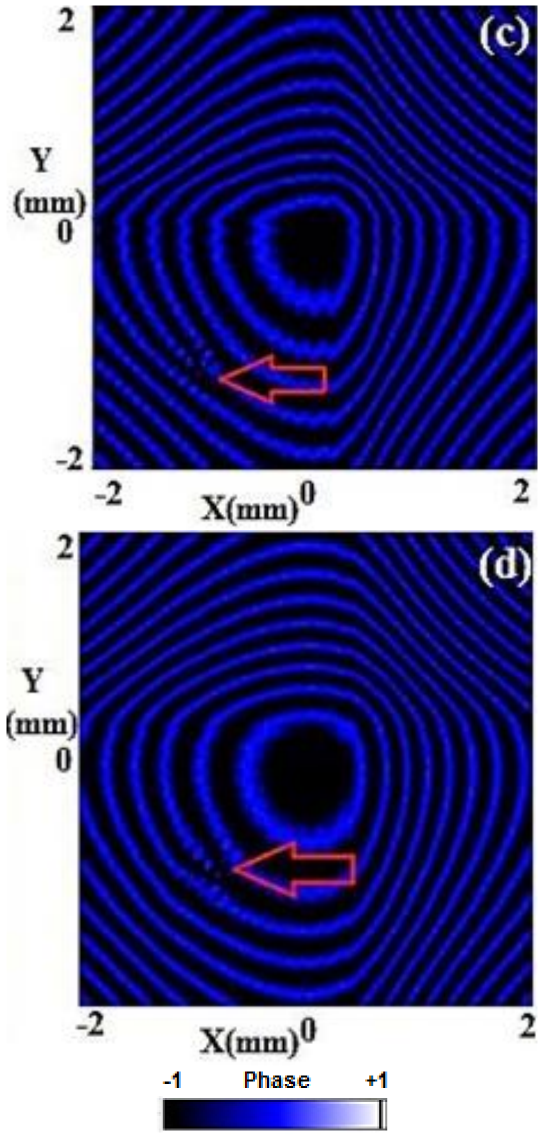
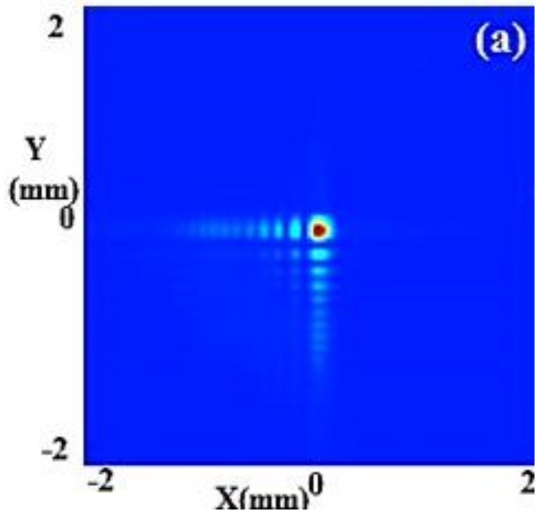


Fig. 8. Phase pattern of the AiB with OV for p=0.5 at; (a) z=0.4d; (b) z=0.5d, (c) z=0.6d, (d) z=0.8d.



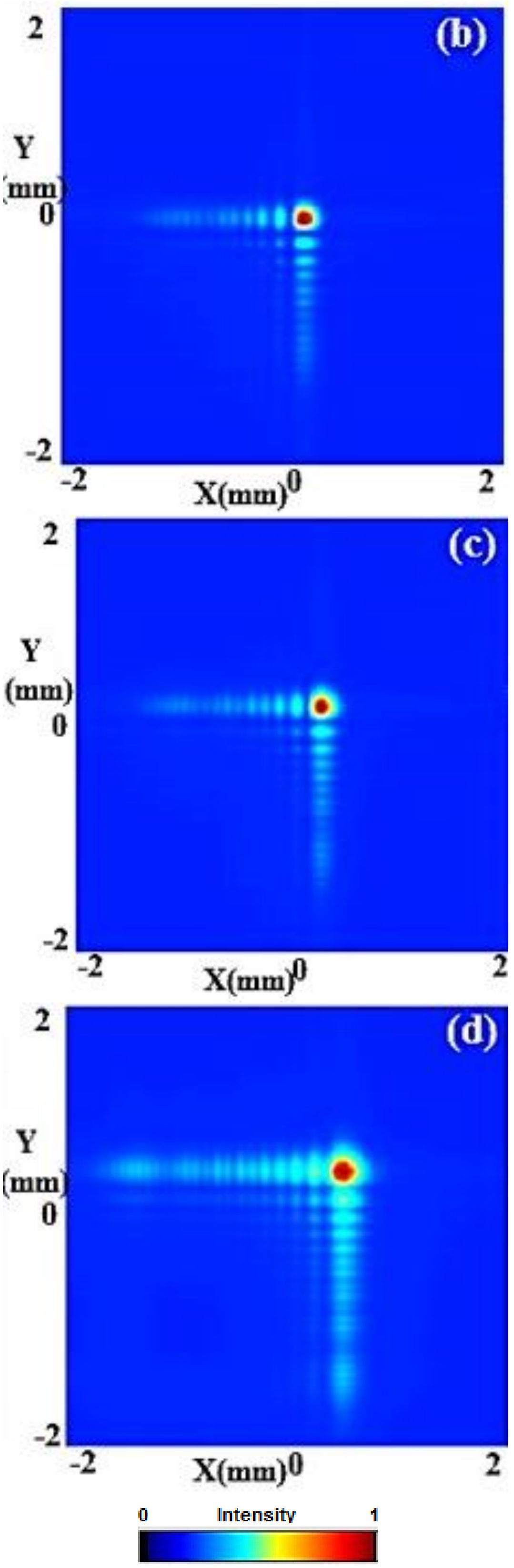


Fig. 9. Intensity pattern of the AiB with OV for p=0.5 at; (a) z=0.4d; (b) z=0.5d, (c) z=0.6d, (d) z=0.8d.

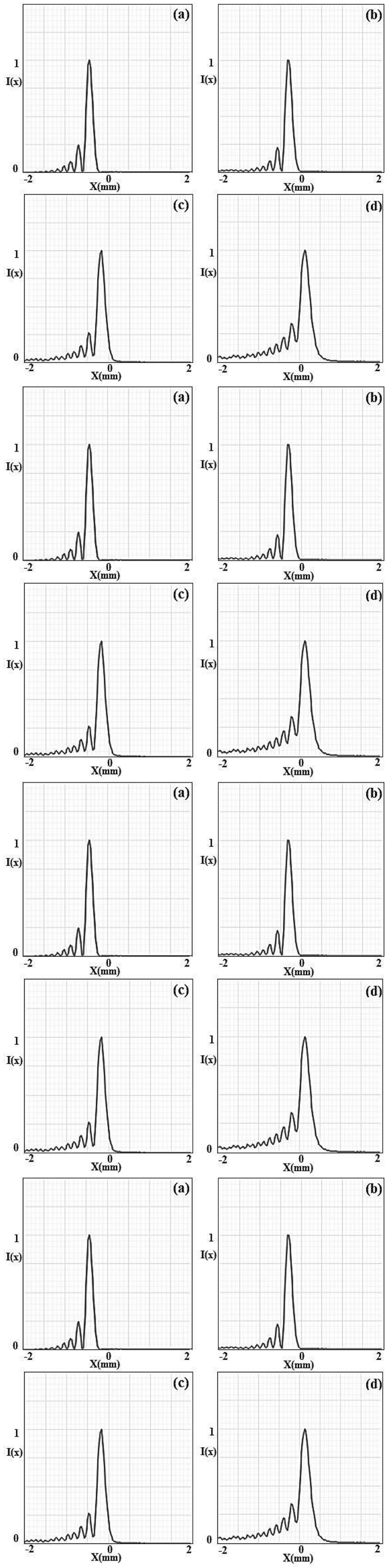


Fig. 10. Intensity normalized in the x- direction of AiB with OV for p=0.5 at; (a) z=0.4d; (b) z=0.5d, (c) z=0.6d, (d) z=0.8d.

Figure 10 shows the two-dimensional intensity graphs of Eq. (8) for different *z*. In Fig. 10, the lateral side lobs will be far from the *x* axis when the value of *z* increases.

Figure 11 shows the two-dimensional spot beam size (S) graph of the AiB with OV for different z. With increasing the value of z, the main peak expands.

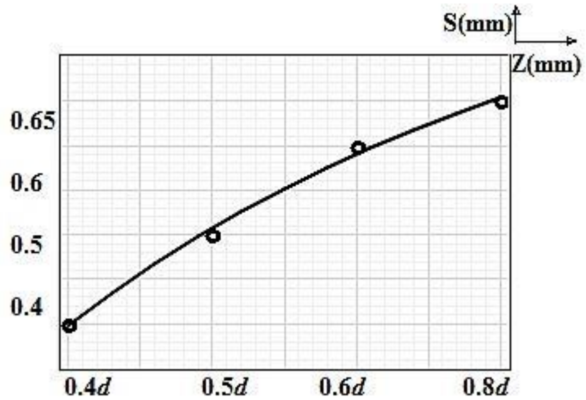


Fig. 11. Spot beam size (*S*) of the AiB with OV for different distance (*Z*).

IV. Conclusion

The new results and the results obtained earlier revealed that, the order of fractional FT *p* not only affects the beam spot size of an AiB, but also controls the orientation of the beam spot. The position of the OV in an optical system changes with changing the power of the fractional FT (*p*). In phase pattern, the process of *p* from 0 to 1, the position of the OV is closer to the center. Also, when the power of the fractional FT (*p*) is changed, the patterns of intensity and phase also change. In the intensity pattern of AiB, there is a uniform increase in the height of the peaks. Lobs of the AiB with OV are destroyed. With *p* tending towards 1, the main peak straitens and the number of spots increases. With increasing the value of *z*, the main peak expands and the effective beam size increases. If the value of *p* is closer to 1 and the value of *z* increases, the lateral side lobs will be far from the *x* axis. When $p<1$, the beam spot of the AiB decreases with increasing value of *p*, and when $1<p<2$ the beam spot of the AiB increases with increasing value of *p*. It was concluded that the behavior of the AiB with OV controls changes the order of the fractional FT in the optical system.

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