Characteristics of the Temporal Behavior of Entanglement between Photonic Binomial Distributions and a Two-Level Atom in a Damping Cavity

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ABSTRACT— In the present study, temporal behavior of entanglement between photonic *binomial* distributions and a two-level atom in a leaky cavity, in equilibrium with the environment at a temperature T, is studied. In this regard, the master equation is solved in the secular approximation for the density matrix, when the initial photonic distribution is binomial, while the atomic states obey the Boltzmann distribution. The atom-photon density matrix so calculated is then used to compute the negativity, as a measure of entanglement. The behavior of atom-photon entanglement is, consequently, determined as a function of time *and* temperature. To justify the behavior of atom-photon entanglement, moreover, we employ the total density matrix to compute and analyze the time evolution of the initial photonic binomial probability distribution. Our results, along with representative figures reveal that the atom-photon degree of entanglement exhibits oscillations while decaying with time and asymptotically vanishes. It is further demonstrated that an increase in the temperature gives rise to a decrease in the entanglement. The finer characteristics of the temporal behavior of the corresponding probability distribution and, consequently, the atom-photon entanglement is also given and discussed.

KEYWORDS: Atom-photon entanglement, Cavity damping, Master equation, Photonic binomial distribution.

I. INTRODUCTION

Quantum entanglement [1] has been by now recognized as one of the most fundamental and intriguing features of composite quantum systems. In fact, quantum entanglement has been employed to develop new means of processing [2], storage [3], transmission [4], etc. [5], [6] of information. It is therefore of great interest to find physical systems where the entanglement can be generated, manipulated and controlled. In the last decades, there have been different proposals to produce entangled states, such as those based on atomic systems [7], quantum electrodynamics cavities [8] and atom-photon interactions [9], [10]. The vast interest in the entanglement of atoms and photons stems from the fact that the former are reliable units for long time storage and processing of information [11], while the latter form rapid carriers of quantum information [12]. In addition, generation of atom-photon entanglement has been experimentally implemented through interaction of a single photon with a trapped atom [13]. What is missing in these treatments, however, is the effect of the environment, modeled as a lossy cavity, on the temporal behavior of entanglement at any temperature. It is therefore the main purpose of the present report to combine the notions of Jaynes-Cummings model (JCM) [14], [15], cavity damping [16]-[18] and negativity [19] to investigate atom-photon entanglement. In the present treatment, however, the so-called *phase damping* which is

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responsible for the decay of atomic states [20] is ignored. To be more specific, in our treatment it is assumed that the photons are statistically binomial [21], while the atoms obey the Boltzmann distribution [22]. As it is well known, the JCM, often expressed in the rotating wave approximation, adequately describes the interaction between a single mode quantized electromagnetic field and a two-level atom. As we have already mentioned, the atom-photon interaction normally takes place in a lossy cavity, so that the state of the total system has to be described by a mixed density operator [23] which satisfies the master equation [24]. The solution of the master equation may then be employed to extract any desired information, in particular, the degree of atom-photon entanglement [25].

As is well established, to quantify the entanglement between elements of a composite system in a mixed ensemble, the measure of negativity, derived from the partially transposed density matrix, is most appropriate [26], [27]. In this paper, therefore, the negativity is used to study the temporal behavior of entanglement between a two-level atom and a binomial photonic distribution, based on the JCM, in a dissipative cavity. The cavity is assumed to be in thermal equilibrium with the environment at a temperature T. The manner of solving the corresponding master equation for the total density operator is introduced and used to determine the evolution of the initial photonic binomial distribution as well as the negativity. A brief account of the results shall appear in the next paragraph where the organization of the article is presented.

The remainder of this article is organized as follows. In section II, the physical model and the corresponding master equation for the density matrix are presented. In section III we discuss the manner of solving the corresponding master equation in the *secular approximation*, providing an efficient algorithm to evaluate the negativity, as functions of time and temperature. To this end, the initial state of the total system is also presented and discussed in this section. The results of our numerical calculation for the time evolution of the initial *binomial* photonic distribution, as well as the negativity, are presented in section IV. In this section we also present illustrative figures from which one can easily analyze the effects of temperature on the entanglement. Consequently, a discussion of the temporal behavior of atom-photon entanglement is also provided in this section. Along these lines, we also furnish a discussion on the atom-photon entanglement in the extreme limits of the binomial distributions, namely, the Fock and coherent states. Finally, in section V, the main features of the article are summarized and some concluding remarks are made.

II. THE PHYSICAL MODEL AND MASTER EQUATION

The system under consideration here consists of a single-mode quantized field of angular frequency ω, interacting with a two-level atom, inside a dissipative cavity, in equilibrium with the environment at a temperature, T. The atomic ground and excited states are denoted by $|g\rangle$ and $|e\rangle$, respectively. In this model, the photons dissipate to the environment through imperfect conducting cavity walls. To account for the field dissipation, it is customary to assume that the walls of cavity form a reservoir of simple harmonic oscillators which, in turn, is minimally coupled to the field. Eliminating the reservoir contributions from the dynamical equation for the total density operator, by partial tracing over the reservoir states, one obtains the master equation for the atom-field system at a temperature, T, as [22], [24],

$$\begin{aligned}\dot{\rho} &= -\frac{i}{\hbar}[H_I, \rho] \\ &\quad -\frac{\gamma}{2}\bar{n}_{th}(aa^{\dagger}\rho - 2a^{\dagger}\rho a + \rho aa^{\dagger}) \\ &\quad -\frac{\gamma}{2}(\bar{n}_{th}+1)(a^{\dagger}a\rho - 2a\rho a^{\dagger} + \rho a^{\dagger}a),\end{aligned} \tag{1}$$

where a ($a^{\dagger}$) denotes the photonic annihilation (creation) operator and ρ, the density operator for the total system of atom-photon is in the interaction picture. The decay constant, γ, is

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related to the quality factor of the cavity Q, by the relation $\gamma = \omega / Q$ [22] and $\bar{n}_{th} = \frac{1}{e^{\mathrm{h}\omega/kT} - 1}$ is the number of thermal bosons of the reservoir at a temperature T, where h and k are the Planck and Boltzmann constants, respectively. For simplicity, the field is assumed to be in resonance with the atoms, $(\omega = \omega_0)$ and thus the atom-photon interaction Hamiltonian in the interaction picture is given by,

$$H_I = \mathrm{h}\lambda\,(a|e\rangle\langle g| + a^{\dagger}|g\rangle\langle e|), \tag{2}$$

where λ determines the atom-photon coupling strength. To avoid a misconception that may arise when the secular approximation (see the discussion right after Eq. (7)) is employed, we call attention to the essence of the rotating wave approximation in which *the energy non-conserving operators are disregarded* [28]. The manner of solving Eq. (1), with the interaction given in Eq. (2) is the subject of next section.

III. ATOM-PHOTON ENTANGLEMENT

In this report, the measure of negativity, defined as, [19], [27],

$$E_N = \sum_i \max(0, -\lambda_i), \tag{3}$$

is used to investigate the atom-photon entanglement. The λ_is in Eq. (2) are the eigenvalues of the partially transposed density matrix, denoted by ρ^{PT}. At this point it is worth mentioning that the transposition can be made relative to any one of the subsystems [19]. The state of the composite system is necessarily entangled if $E_N > 0$ but it may or may not be entangled if $E_N = 0$ [19].

In this paper, we assume that the atom is initially in thermal equilibrium with the cavity at the temperature T. Therefore, the atomic state is distributed among the ground and excited states, determined by the Boltzmann factor,

$$\begin{aligned}\rho_A(0) &= \frac{e^{\mathrm{h}\omega/2kT}}{2\cosh(\mathrm{h}\omega/2kT)}|g\rangle\langle g| \\ &\quad + \frac{e^{-\mathrm{h}\omega/2kT}}{2\cosh(\mathrm{h}\omega/2kT)}|e\rangle\langle e| \\ &\equiv A(T)|g\rangle\langle g| + B(T)|e\rangle\langle e|.\end{aligned} \tag{4}$$

On the other hand, we assume that initially the photons are prepared with a binomial distribution, using the experimental setup presented in [29], then delivered into the cavity via a very small aperture. Then the photonic initial state is,

$$\rho_F(0) = |p, M\rangle\langle p, M|, \tag{5}$$

where the binomial state, $|p, M\rangle$, is an expansion in terms of Fock states, $\{|n\rangle\}$, as,

$$\begin{aligned}|p, M\rangle &= \sum_{n=0}^{M}\left[\frac{M!}{n!(M-n)!}\,p^n(1-p)^{M-n}\right]^{\frac{1}{2}}|n\rangle \\ &\equiv \sum_{n=0}^{M} C_n|n\rangle.\end{aligned} \tag{6}$$

In Eq. (6), $0 < p < 1$ represents the probability of occurring a photon in the expansion, while M denotes the maximum photon number of the binomial state [21]. The mean photon number is then given by $\bar{n} = pM$. From the definition, it is clear that the binomial state describes a photonic state between two extremes of a Fock state and a coherent one. In fact, the number state of M (zero) photons results when, $p \to 1(0)$, $|p, M\rangle \to |M\rangle\,(|0\rangle)$, while the coherent state is the result of letting, $p \to 0,\ M \to \infty$ in Eq. (6) [21]. Parenthetically, we mention that a thorough examination of the entanglement between atoms and a photonic *coherent* state, in a lossy cavity, is made in Ref. [30].

The state of the composite system is initially given as,

$$\rho(0) = \rho_A(0) \otimes \rho_F(0), \tag{7}$$

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which describes a pure and separable state. In order to calculate the negativity, the first step is to perform the unitary transformation, $W(t)=e^{iH_I t/\mathrm{h}}\rho(t)\,e^{-iH_I t/\mathrm{h}}$, on Eq. (2), giving a simpler equation for $W(t)$. In fact, when use is made of the bases of H_I and assuming a weak, relative to the atom-photon coupling, damping such that $\lambda ? \gamma$ (the well-known secular approximation [31]), the set of coupled equations for the transformed density matrix elements separates into a set of independent equations for the off-diagonal elements of $W(t)$. The set of equations for the off-diagonal elements of $W(t)$ may be analytically solved, straightforwardly. Although we have used these solutions in the calculation of our final results, for the sake of brevity, we do not present them here. The off-diagonal elements of the original density operator then read,

$$\begin{aligned}&\langle g,m+1|\rho(t)|g,n+1\rangle = \\ &e^{-\frac{\gamma}{2}t(m+n+1)}\;e^{-\gamma\bar{n}_{th}t(m+n+2)} \\ &\Big[AC_{m+1}C_{n+1}^{*}\cos\left(\lambda t\sqrt{m+1}\right)\cos\left(\lambda t\sqrt{n+1}\right)+ \\ &\;BC_m C_n^{*}\sin\left(\lambda t\sqrt{m+1}\right)\sin\left(\lambda t\sqrt{n+1}\right)\Big],\end{aligned} \tag{8}$$

$$\begin{aligned}&\langle g,m+1|\rho(t)|e,n\rangle = i\,e^{-\frac{\gamma}{2}t(m+n+1)}e^{-\gamma\bar{n}_{th}t(m+n+2)} \\ &\Big[AC_{m+1}C_{n+1}^{*}\cos\left(\lambda t\sqrt{m+1}\right)\sin\left(\lambda t\sqrt{n+1}\right)- \\ &\;BC_m C_n^{*}\sin\left(\lambda t\sqrt{m+1}\right)\cos\left(\lambda t\sqrt{n+1}\right)\Big],\end{aligned} \tag{9}$$

$$\begin{aligned}&\langle e,m|\rho(t)|e,n\rangle = e^{-\frac{\gamma}{2}t(m+n+1)}e^{-\gamma\bar{n}_{th}t(m+n+2)} \\ &\Big[AC_{m+1}C_{n+1}^{*}\sin\left(\lambda t\sqrt{m+1}\right)\sin\left(\lambda t\sqrt{n+1}\right)+ \\ &\;BC_m C_n^{*}\cos\left(\lambda t\sqrt{m+1}\right)\cos\left(\lambda t\sqrt{n+1}\right)\Big],\end{aligned} \tag{10}$$

$$\begin{aligned}&\langle g,0|\rho(t)|g,n+1\rangle = e^{-\frac{\gamma}{2}t\left(n+\frac{1}{2}\right)}e^{-\gamma\bar{n}_{th}t} \\ &\qquad AC_0C_{n+1}^{*}\cos\left(\lambda t\sqrt{n+1}\right),\end{aligned} \tag{11}$$

$$\begin{aligned}&\langle g,0|\rho(t)|e,n\rangle = i\,e^{-\frac{\gamma}{2}t\left(n+\frac{1}{2}\right)}e^{-\gamma\bar{n}_{th}t} \\ &\qquad AC_0C_{n+1}^{*}\sin\left(\lambda t\sqrt{n+1}\right),\end{aligned} \tag{12}$$

In Eqs. (8) through (12), $m,n=0,1,2,...$ and $m\neq n$. On the other hand, the diagonal elements satisfy a set of $M+2$ coupled, recursive first-order differential equations. To the best of our knowledge, there is no analytical solution for the diagonal elements and we have to employ numerical methods. It is worth mentioning that the foregoing procedure is quite general and can be applied to any such systems [30], the difference being in the specification of the initial states. Having determined the elements of the original density matrix, we can partially transpose it, relative to one of the subsystems, with ease. Moreover, the large dimensions of the density matrix force us, again, to resort to numerical calculation of its eigenvalues. We emphasize that the resulting density matrix straight forwardly gives the time evolution of any initial quantity, photonic, atomic or the composition, for the binomial probability distribution. Related to the present investigation, and as an example, one can easily compute the time evolution of the initial photonic probability distribution, $P_n(t,T)=Tr[|n\rangle\langle n|\rho(t,T)]$. In the next section these points are used to investigate the time evolution of the binomial distribution as well as the behavior atom-photon entanglement.

IV. Numerical Results and Discussion

Having described the manner of calculating the density matrix for the composite atom-photon system (and its partially transposed one) we are now in the position of investigating the behavior of the photonic probability distribution and the corresponding entanglement. The result of our calculation of the evolution of the initial probability distribution is illustrated in Fig. 1. In this typical figure and the following ones we have set the field frequency at $\omega=1$ THz, the damping constant, $\gamma=0.03\lambda$ and time is scaled as, $\tau=\lambda t$. In this particular figure the temperature

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is taken as, $T = 4K$ and the initial photon mean number as, $\bar{n} = 2$. The probability of occurrence of one photon when *M* photons are already present, *p*, can be experimentally measured [29] and, accordingly, we have chosen it as $p = 1/3$. This choice then selects the limit of summation in Eq. (6) from $M = \bar{n}/p$.

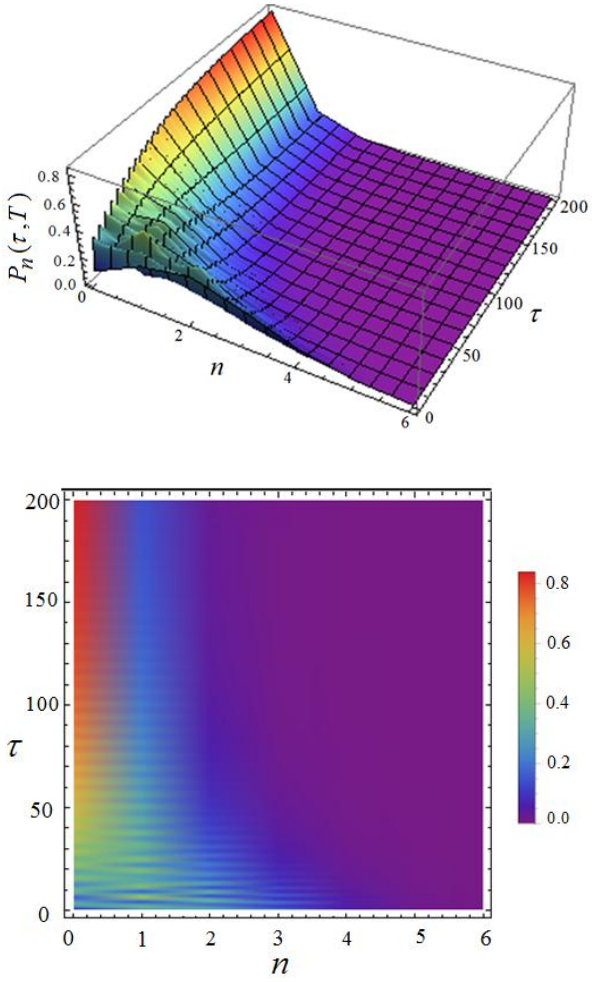


Fig. 1. Photonic probability distribution, $P_n(\tau, T)$ versus n and scaled time, τ, along with the corresponding density plot, at a fixed temperature of $T = 4K$.

From this figure it is evident that the probability of any photonic state, except the vacuum, is a descending function of time and vanishes asymptotically. The diminishingly small initial probability of no photon in the distribution, however, is an ascending function of time and approaches unity asymptotically. This observation suggests that after sufficiently long time the field falls into its vacuum state, an indication of separability of atom-photon states. In correspondence to this figure, we illustrate the behavior of negativity, as a measure of atom-photon entanglement, in Fig. 2.

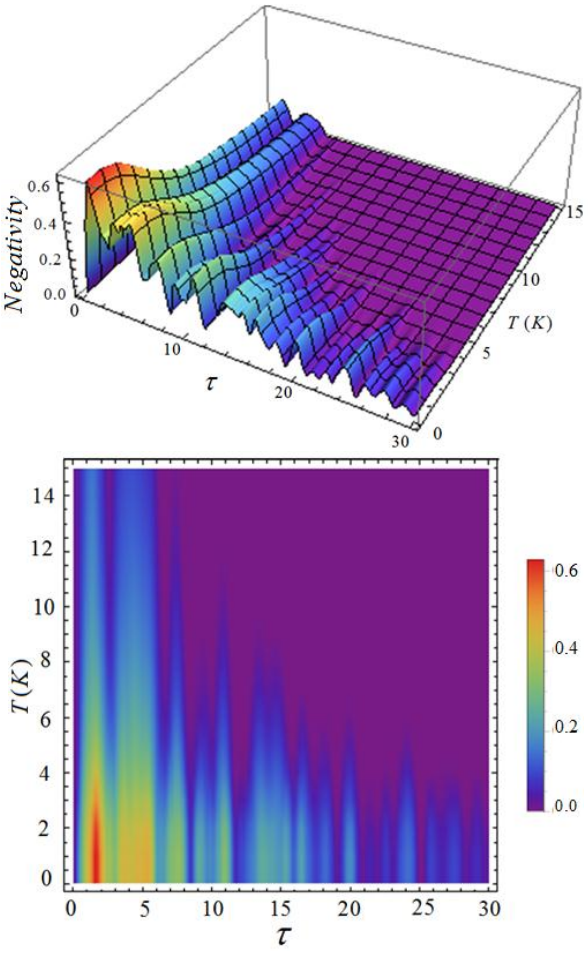


Fig. 2. Negativity versus scaled time, τ and temperature, *T*, along with the corresponding density plot.

Such general features of atom-photon entanglement are attributed to the physical fact that atoms and photons periodically exchange energy, leading to oscillations. Meanwhile, the field energy is gradually lost through the dissipative cavity, giving rise to a decaying oscillation in the temporal behavior of the entanglement. We further observe that the degree of entanglement, while oscillating, grows at earlier times reaching its maximal value. This is due to the fact that as the atoms start interacting with the reservoir modes, the ensemble tends towards a mixed one. On the other hand, an increase in the temperature results a reduction in the maximal atom-photon entanglement at any time. The atom-photon

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entanglement then disappears as temperature rises. This demeanor is physically due to the fact that an increase in the temperature gives rise to the excitation of more atom-photon states with almost equal probabilities. In the limit of extremely high temperatures, they become exactly equal and the system approaches a fully mixed state, with no entanglement [32]. For illustration of the finer detail of entanglement between atoms and binomial photonic states in a lossy cavity, we present Figs. 3 and 4. The probability of photon occurrence in these two figures is again taken as $p = 1/3$. Figure 3 is devoted to the time (scaled) variation of negativity for different initial photon mean number at a fixed temperature of $4K$, while in Fig. 4 the same is done for various temperature and a fixed initial photon mean number of $\bar{n} = 2$.

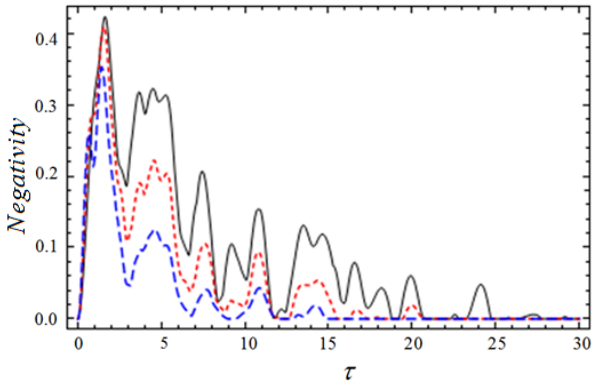


Fig. 3. Time evolution of negativity for initial mean photon numbers of $\bar{n} = 2$ (solid curve, black), $\bar{n} = 3$ 3 (small-dashed curve, red) and $\bar{n} = 4$ (large-dashed curve, blue).

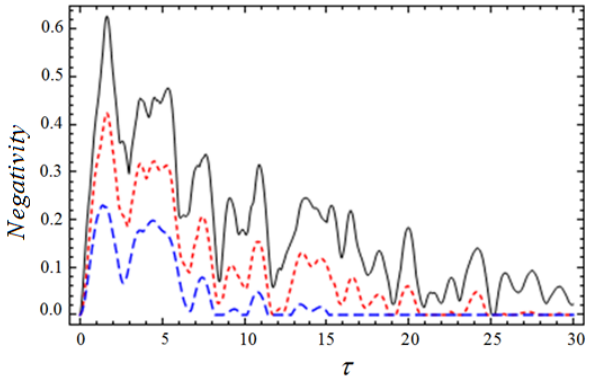


Fig. 4. Time evolution of negativity at temperatures of $T = 0K$ (solid curve, black), $T = 4K$ (small-dashed curve, red) and $T = 8K$ (large-dashed curve, blue) for the initial mean photon number of $\bar{n} = 2$.

From Fig. 3 it is evident that for a lower initial mean photon number the system exhibits a larger degree of entanglement while diminishes at a slower pace. On the other hand, Fig. 4 indicates that the degree of atom-photon entanglement is by far larger and is retained for a longer time as the temperature is decreased.

It is also of interest to examine the limiting cases of the binomial distribution, as given in Eq. (6) and discussed right after wards. To this end, we set p ; 1 in our numerical computation to study the atom-photon entanglement when the photon state is initially a Fock one. The result is depicted in Fig. 5.

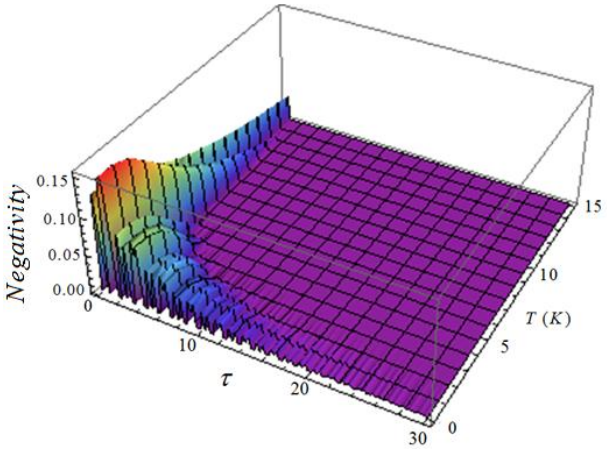


Fig. 5. Negativity versus scale time, τ, and temperature, T, for the case of an initial photonic Fock state, $|6\rangle$.

Again, the general reasoning for the behavior of atom-photon entanglement, as we discussed earlier, still holds. However, in this case the degree of entanglement reduces and vanishes more rapidly, due to the presence of just one photonic state initially. Moreover, for the other extreme case of photonic vacuum state, $|0\rangle$, we set p ; 0 in our calculations. The result is illustrated in Fig. 6. We observe that at earlier times an increase in the temperature leads to a decrease in the degree of entanglement, reaching a steep dip, followed by an increase and asymptotically vanishes. Even though the same lines of argument also apply here, we suffice to elaborate on the presence of the dip. This phenomenon is due to the fact that at earlier times and absolute zero temperature, the system is in a pure entangled state. As the

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temperature rises the ensemble becomes a mixed but separable one. At higher temperature, however, the ensemble turns into a mixed entangled one and ends up in a fully mixed state. As is expected, the degree of entanglement is much lower than the binomial case.

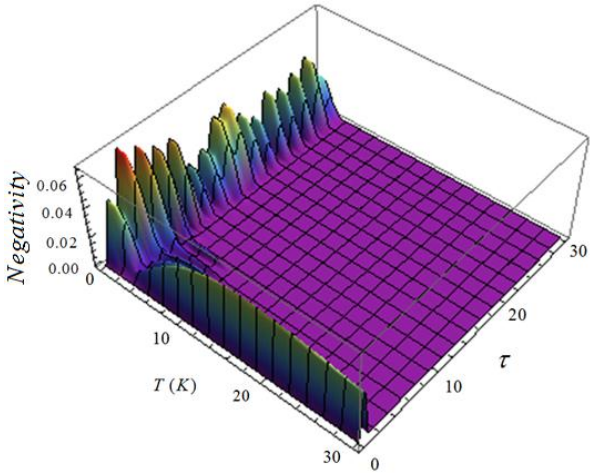


Fig. 6. Negativity versus scaled time, τ, and temperature, T, for the case of an initial photonic Fock state, $|0\rangle$.

We end this section by presenting the behavior of atom-photon entanglement for the other limiting case of the binomial distribution, namely, the photonic coherent state. This aim is achieved by letting $p = 0.01$ and $M = 200$ (giving a photon mean number of $\bar{n} = 2$) in our calculations. The atom-photon degree of entanglement, as a function of time and temperature, is illustrated in Fig. 7.

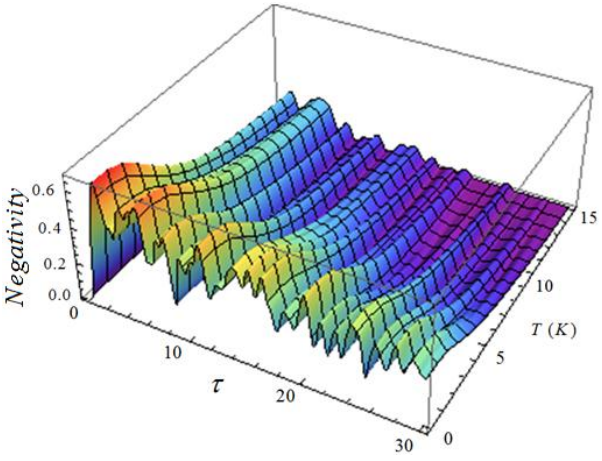


Fig. 7. Negativity versus scaled time, τ, and temperature, T, for the case of an initial photonic *coherent* state with initial mean photon number of $\bar{n} = 2$.

From Fig. 7 it is noticed that although the maximal degree of entanglement is practically the same as the binomial distribution, the totally mixed state is reached at higher temperatures. Moreover, it takes a much longer time for the entanglement to disappear in the coherent case, as compared to the binomial distribution. This observation is again due to the fact that more photonic states participate in the entanglement when the photonic state is a coherent one [30].

V. SUMMARY AND EPILOGUE

The present report is devoted to the study of the entanglement for a mixed ensemble of two-level atoms and photons in a lossy cavity. The mixture is specified by the atomic Boltzmann distribution at an equilibrium temperature, T, while the photons are externally injected into the cavity with binomial distribution. Since the ensemble is a mixed one, we take advantage of the concept of negativity to determine the temporal behavior of the degree of atom-photon entanglement. To this end, we first demonstrate how the master equation, involving the Jaynes-Cummings model and cavity damping, is solved for the total density matrix. From the so computed total density matrix we then determine the time evolution of the initial probability distribution as well as the degree of atom-photon entanglement. As the limiting extremes of the binomial photonic distribution, we have also examined the atom-photon entanglement for a coherent and Fock states. Even though a full discussion of the results is presented in the main body of the article, in what follows we outline the more important points.

- At every temperature, the atom-photon degree of entanglement starts from zero, oscillates at short times reaching a notable maximum.

- The entanglement diminishes as time passes and asymptotically vanishes. As a result, information is periodically

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shared by atoms and photons and eventually becomes local (separable).

- Our analysis also quantitatively confirms the well-known fact that, at a fixed time, an increase in the temperature reduces the maximal value of atom-photon entanglement and disappears at higher temperatures. This point is in conformity with the fact that at high enough temperatures information becomes localized.

- When the photonic initial state is a single Fock state, the degree of atom-photon entanglement is drastically reduced.

- As for the case of coherent initial state, the maximal degree of entanglement is almost the same as in the case of binomial distribution. The manner of approaching separability, both in time and temperature, is quite different for the two cases.

As a final point, it is worth mentioning that with a setup similar to the one given in [33], one can, in fact, make a measurement on the atomic state. Meanwhile, a photon detector, placed properly, can specify the photonic states. The results of the two measurements indeed give an indication of atom-photon entanglement.

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