Voltage-Controlled Entanglement between Quantum-Dot Molecule and its Spontaneous Emission Fields via Quantum Entropy

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ABSTRACT— The time evolution of the quantum entropy in a coherently driven three-level quantum dot (QD) molecule is investigated. The entanglement of quantum dot molecule and its spontaneous emission field is coherently controlled by the gat voltage and the intensity of applied field. It is shown that the degree of entanglement between a three-level quantum dot molecule and its spontaneous emission fields can be decreased by increasing the tunneling parameter.

KEYWORDS: Entanglement, Quantum dots, Quantum entropy

I. INTRODUCTION

Quantum entanglement is a basic concept of the quantum information processes, such as quantum computing [1], quantum teleportation [2], quantum dense coding [3], and quantum cryptography [4]. When two quantum particles interact, they can no longer describe independently of each other. Two or more quantum systems are entangled when it can be described their physical properties by means of a direct product of their respective density operators. A superposition of such products entanglement shared by distance parties is not only a key ingredient for the rest of quantum nonlocality [5], but also is a basic resource in achieving task of quantum communication and quantum computation [6]. Entanglement is a natural consequence of linearity of Hilbert space, its generation and measurement has intensively been proposed [7]. Various quantum systems have been suggested as possible candidates in engineering of quantum entanglement [8-10]. Recent studies have been concentrated in producing entangled state of atoms that include production of maximally entangled state of two two-level atoms [11]. Generation of the entangled pair of atoms was proposed by two micro-maser cavities [12, 13], laser pulse [14], and collision processes [15]. Entanglement of two level atoms [16], quantum entangled states of trapped ion [17], and three photon entangled state have been studied [18]. As far as field states are concerned, various methods for generating photon entangled states are purposed [19-22]. In recent years, much attention has been focused on properties of the entanglement between the field and the atom via the entropy of the Jaynes Cummings model [21-23]. In fact, atom-photon interaction inside the cavity leads to another interesting tool for the entanglement. It is shown that Von Neumann entropy is a very useful operational measure of the parity of the quantum state [24, 25]. The time evolution of the field (atomic) entropy reflects the time evolution of the entanglement between the atom and the field. The higher is the entropy, the greater the entanglement. Thus, the quantum entropy can be used as a measure of the degree of entanglement. Entangled state of the single three-level atom interacting with a single electromagnetic field mode in an ideal cavity, undergoing either one or two-photon transition, has been studied [23]. Furthermore, the evolution of the atomic (field) entropy for the three-level atom interacting with one-mode [26, 27] and two-mode [28] cavity field has also been presented.

In some other studies, intensity dependent entanglement of a V-type three-level atom with a single mode field is proposed [26]. In addition, the entanglement of a ladder type three-level atom interacting with a non-correlated two modes cavity field [29], and entanglement between Λ-type three-level atom and its spontaneous emission fields [30] is also presented. However, ordinary optical transitions and very high finesse cavities make it possible to meet the strong coupling conditions and observe interesting quantum effects. The very fast time evolution of these optical systems has not made it possible so far to investigate entanglement directly. So, the individual systems should be prepared in a well-defined initial quantum state. This should be very well isolated from the environment, and interact strongly with each other, as required for the realization of quantum gates. These states should be accurately detected. Furthermore, individual qubit addressing is required for engineering of the most general entangled state, scalable to arbitrary numbers of qubits, as well as for performing fundamental test of quantum measurement theory. Many proposals have been made to implement these requirements in solid-state devices, mesoscopic conductors [31], photonic band gap crystal [32] and quantum dots [33, 34].Among many proposals, quantum entanglement coupled quantum dot molecule is widely investigated [35]. Qurioga *et al.* [36] showed that exciton in coupled quantum dots are ideal candidates for reliable preparation of entangled states in solid-state systems. Optically controlled exciton dynamics in coupled quantum dots [36], generation of multiple entangled state in a system of N quantum dots embedded in a microcavity [37], and a system of two coupled quantum dots entangled through their interaction with a cavity mode [33] have also been discussed.

In this paper, we consider an asymmetric double quantum dot structure, and investigate the possibility of entanglement between quantum dot and its spontaneous emission fields. We find that the degree of entanglement between quantum dot molecule and its spontaneous emission fields can be controlled by tuning parameters of the system such as the gate voltages and the laser intensity.

Note that the most of previous proposals on quantum entropy and quantum entanglement of the field and atom are proposed in gaseous systems. But, as mentioned, the fast evaluation of gaseous system makes impossible to investigate the steady state quantum entanglement. Here, the quantum entropy as well as quantum entanglement of quantum dot molecule and its spontaneous emission fields is described. Moreover, the entanglement of the quantum dot and its spontaneous emission fields can be employed in quantum information, quantum computing, and quantum teleportation. In addition, the existence of the controlling parameters such as gate voltage and the intensity of applied field can control the quantum entropy of the quantum dot molecule and its spontaneous emission fields.

The paper is organized as fellows: in section 2, we present the model and equations. The relation between entanglement and the van-Neumann entropy is proposed in section 3. The results and discussion is presented in section 4, and the conclusion can be found in section 5.

II. MODEL AND EQUATIONS

Consider an asymmetric double quantum dot structure as shown in Fig. 1. The QD molecule consists of two dots (the left one and right one) system coupled by tunneling. Such a QD molecule can be fabricated using the self-assembled dot growth technology [11]. The asymmetric QD molecule can be viewed in a double layer InAs/GaAs structure [13]. Two levels $|0\rangle$ and $|1\rangle$ are the lower valance and upper conducting band levels of the left QD. However, level $|2\rangle$ is the excited conducting level of the second QD. The schematic representation of the band structure and the level configuration are shown in Fig. 1 (a and b). Without a gate voltage, the levels are out of resonance resulting in a weak inter dot tunneling. In the presence of a gate voltage, the conduction band levels get close to resonance leading to increasing their coupling, while the valance band levels get more off-resonance resulting in effective decoupling of these levels.

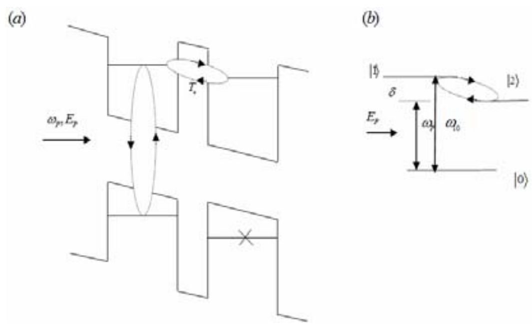

Fig. **1** (a) Band diagram of a three-level QD molecule. It consists of two dots (the left one and the right one). With an external voltage applied to a gate electrode, the conduction-band levels get closer to resonance, greatly increasing their coupling, while the valence-band levels get more off-resonance, resulting effective decoupling of those levels. (b) Schematic of the energy levels arrangement. A probe laser with central frequency ω_p and amplitude E_P excites one electron from the valence band to the conduction band in the left dot, which can in turn tunnel to the right dot.

The diagram of the energy level of the QD molecule after applying the gate voltage is shown in Fig. 1 (b). A weak tunable probe field of frequency ω_p and Rabi- frequency $\Omega = \vec{\wp}_{01}.\vec{E}_p / 2\hbar$ applies to $|0\rangle \leftrightarrow |1\rangle$ transition. Here E_P is the amplitude of the probe field, and $\vec{\wp}_{01}$ represents the electric dipole moment. Note that the probe field can excite one electron from valance band to conducting band in the left QD that can tunnel to the right one. Here, the electromagnetic field couples the ground state $|0\rangle$ (the system without excitations) to the state $|1\rangle$ (a pair of electron and hole bound in the first dot). We neglect the hole tunneling and write the interaction Hamiltonian as:

$$H = \sum_j \hbar\omega_j \, |j\rangle\langle j| + Te(|1\rangle\langle 2| + |2\rangle\langle 1|) + \hbar\Omega(e^{-i\omega_p t}\,|0\rangle\langle 1| + e^{i\omega_p t}\,|1\rangle\langle 0|), \tag{1}$$

where $\hbar\omega_j$ is the energy of the state $|j\rangle$, and Te denotes the electron tunneling matrix element.

The resulting effective Hamiltonian can be written as:

$$H = \begin{pmatrix} -\frac{\delta}{2} & \Omega & 0 \\ \Omega & \frac{\delta}{2} & T_e \\ 0 & T_e & \frac{\delta}{2} - \omega_{12} \end{pmatrix}, \tag{2}$$

where $\hbar = 1$, $\delta = \omega_{10} - \omega_p$ is frequency detuning of the probe laser with the corresponding transition (see Fig. 1 (b)), and $\omega_{ij} = \omega_i - \omega_j$ with ω_j being the frequency of the state $|j\rangle (j = 0,1,2)$. The parameters T_e and ω_{12} can be tuned with bias voltage. The dynamics of the system is described by the density matrix equation of motion

$$\dot{\rho} = -\frac{i}{\hbar}[H, \rho] - 1/2\{\Gamma, \rho\}, \tag{3}$$

where ρ denotes the element of density matrix, and Γ represents the spontaneous decay and dephasing processes.

Substituting equations (2) into equation (3) and after moving to an appropriate rotating frame, we obtain the flowing set of equations for the elements of density matrix

$$\begin{aligned}
\dot{\rho}_{00} &= \gamma_{10}\rho_{11} + \gamma_{20}\rho_{22} + i\,\Omega(\rho_{01} - \rho_{10}) \\
\dot{\rho}_{11} &= -\gamma_{10}\rho_{11} + iT_e(\rho_{12} - \rho_{21}) - i\,\Omega(\rho_{01} - \rho_{10}) \\
\dot{\rho}_{22} &= -\gamma_{20}\rho_{22} + iT_e(\rho_{21} - \rho_{12}) \\
\dot{\rho}_{10} &= -(i\,\delta + \Gamma_{10})\rho_{10} - iT_e\rho_{20} - i\,\Omega(\rho_{00} - \rho_{11}) \\
\dot{\rho}_{20} &= -\left[i(\delta - \omega_{12}) - \Gamma_{20}\right]\rho_{20} - iT_e\rho_{10} + i\,\Omega\rho_{21} \\
\dot{\rho}_{12} &= -(i\,\omega_{12} + \Gamma_{12})\rho_{12} - iT_e(\rho_{22} - \rho_{11}) - i\,\Omega\rho_{02}
\end{aligned}, \tag{4}$$

where $\rho_{ij} = \rho^*_{ji}$ and $\rho_{00} + \rho_{11} + \rho_{22} = 1$. Here $\gamma_{10}(\gamma_{20})$ is the spontaneous emission field from upper level $|1\rangle(|2\rangle)$ to the lower level

$|0\rangle$. Also, Γ_{ij} are dephasing broadening of the corresponding transition.

III. ENTANGLEMENT AND VAN-NEUMANN ENTROPY

Despite the possibility of quantum entanglement was acknowledged almost as soon as quantum theory was discovered, mathematical methods to quantify entanglement has been given only in the last few years. In the case of pure quantum state for two subsystems, a number of physically intuitive measures of entanglement have been known for some time. However, for general mixed state of an arbitrary number of subsystems, entanglement measurement is still under development. In this paper, we use the reduced quantum entropy as a measurement of the degree of entanglement between the spontaneous emission fields and the three-level quantum dot molecule.

The entropy of the quantum dot molecule and the spontaneous emission fields can be defined through their respective reduced-density operators by [24, 25]:

$$S_i(t) = -Tr_i(\rho_i^I \ln \rho_i^I), (i = d, f). \tag{5}$$

The entropy of a general two-component quantum systems are linked by a remarkable theory presented by Araki and Lieb [38]:

$$|S_d(t) - S_f(t)| \le S_{df} \le S_d(t) + S_f(t), \tag{6}$$

where $S_{df} = -Tr(\rho_{df}^I \ln \rho_{df}^I)$ is the total entropy of the quantum dot-spontaneous emission fields system. We note that the $\rho_{df}^I(t)$ given by Eq. (6) is governed by a unitary time evolution, and consequently the total entropy S_{df} is time independent. Since we assume that the quantum dot and the vacuum fields are initially in a disentangled pure state, so the total entropy of the quantum dot-spontaneous emission fields system is zero. One immediate consequence of this assumption is $S_d(t) = S_f(t)$. Consequently, we only need to calculate the V-type three level quantum dot entropy $S_d(t)$. The reduced entropy of the QD moleculecanbe definedthrough their respective reduced- density operators by [24, 25]:

$$S_d(t) = -Tr(\rho_d \ln \rho_d). \tag{7}$$

Here, ρ_d is the reduced density operator of the three-level QD molecule with the elements given in equation (4), and we have set Boltzmann constant equal to 1. We can express the atomic quantum entropy in terms of the eigen-values $\lambda_d(t)$ of reduced density

$$S_d(t) = -\sum_{d=1}^{3} \lambda_d(t) \ln \lambda_d(t). \tag{8}$$

Note that equation 7 or 8 along with equation 4 should numerically be solved to reach the quantum entropy. So, the quantum entropy dependsonthe atomic parameter via equation 4.

IV. RESULTS AND DISCUSSION

We numerically calculate the entanglement between the quantum dot molecule and the spontaneous emission fields via equation (4) and (8). In following results, we choose a typical decay rate $\gamma = 2\pi \times 9.79 meV$, and all the figures are plotted in the unit of γ. In all figures the system initially is prepared in state $|1\rangle$. The influence of the Rabi- frequency of applied fields, i.e. intensity of laser field, in entanglement of the quantum dot molecule and spontaneous emission fields is displayed in Fig. 2. The selected parameters are: $\gamma_{10} = 0.554 meV$, $\gamma_{20} = 0.001 meV$, $T_e \approx 0$, $\Gamma_{10} = 0.001 meV$, $\Gamma_{20} = 0.005 meV$, $\delta = 0$, $\Gamma_{12} = 2 meV$, and $\omega_{12} = 0.2 meV$

Figure 2 shows that quantum entropy quickly rises from zero and reaches to a fixed value as time increases. By increasing the Rabi-frequency from 0.1 *meV* (solid line) to 0.5 *meV* (dashed line) (or 1 *meV* (dotted line)) the steady state quantum dot entanglement increases. Therefore, the degree of entanglement between quantum dot molecule and its spontaneous emission fields will be increased by increasing the Rabi-frequency of applied field.

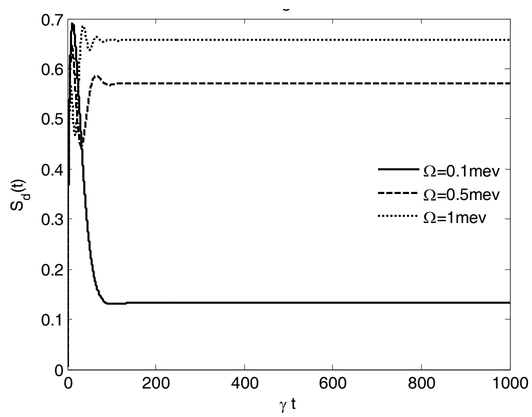

Fig. 2 Evolution of the entanglement as a function of normalized time γt for three different values of Ω. Other parameters are $\gamma_{10} = 0.554 meV$, $\gamma_{20} = 0.001 meV$, $T_e = 0$, $\Gamma_{10} = 0.001 meV$, $\Gamma_{20} = 0.005 meV$, $\delta = 0$, $\Gamma_{12} = 2 meV$, and $\omega_{12} = 0.2 meV$

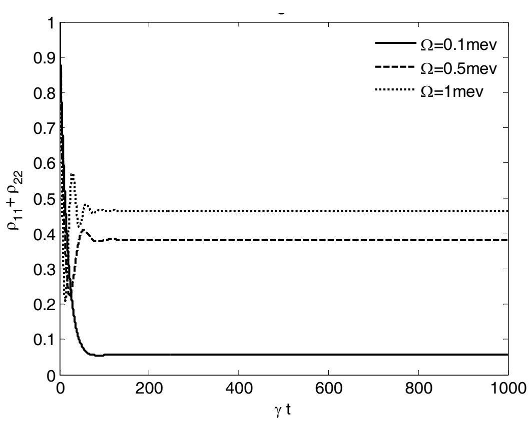

Fig. 3 Evolution of the upper levels population as a function of normalized time γt for three different values of Ω. Other parameters are $\gamma_{10} = 0.554 meV$, $\gamma_{20} = 0.001 meV$, $T_e = 0$, $\Gamma_{10} = 0.554 meV$, $\Gamma_{20} = 0.005 meV$, $\delta = 0$, $\Gamma_{12} = 2 meV$, and $\omega_{12} = 0.2 meV$

To understand the physical mechanism of this behavior, the evolution of upper level population as a function of normalized time γt is displayed in Fig. 3. It can clearly be observed that the upper level population will be increased by increasing the Rabi-frequency of applied field. This may lead to a large entanglement of quantum dot molecule and its spontaneous emission fields.

We are interested in the effect of interdot tunneling on quantum dot-spontaneous emission fields entanglement. We assume that the gate voltage is applied to the quantum dot molecule that induces the tunneling effect. The coherence in this case is created by coupling states via tunneling instead of optical pumping [39]. We note that in gas system the coherence is created by the laser field, while in QD molecule it is created by tunneling effect. The effect of tunneling effect, i.e. T_e, on evolution of quantum dot entanglement and its spontaneous emission fields is displayed in Fig. 4. The tunneling parameters are $T_e = 0.1$ (solid line), 0.5 (dashed line), and 1 (dotted line). Figure 4 shows that by increasing the tunneling effect the QD molecule entropy decreases, leading to reduction of quantum entanglement between quantum dot molecule and its spontaneous emission fields. This is in a good agreement with the result of ref. [40]. In fact, by taking into account the interdot tunneling effect, the probe absorption around zero probe field detuning will be cancelled. This may lead to reduction of population in upper levels leading to spontaneous emission cancellation. Therefore, the entanglement of QD molecule and its spontaneous emission fields will be reduced.

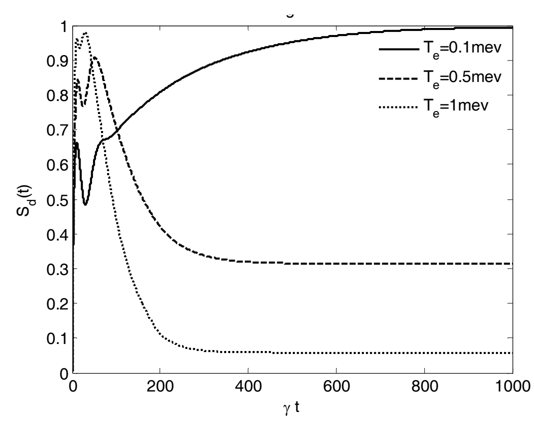

Fig. 4 Evolution of the entanglement as a function of normalized time γt for three different values of T_e. Other parameters are $\gamma_{10} = 0.554 meV$, $\gamma_{20} = 0.001 meV$, $\delta = 0$, $\Gamma_{10} = 0.554 meV$, $\Gamma_{20} = 0.005 meV$, $\Gamma_{12} = 2 meV$, $\Omega = 0.5 meV$, and $\omega_{12} = 0.2 meV$.

Now, we show the effect of probe field detuning on entanglement of the QD molecule and its spontaneous emission fields. We display the steady state entropy S_d as a function of probe detuning δ in Fig. 5. It is obvious that the quantum dot molecule and its spontaneous emission fields are approximately disentangled for $\delta = 0$ and $T_e = 1meV$ (dotted line).

However, for $T_e = 0.1meV$ (solid line) the quantum dot molecule and its spontaneous emission fields are entangled around zero probe field detuning. So, the degree of entanglement between QD molecule and its spontaneous emission fields reduces by increasing the interdot tunneling.

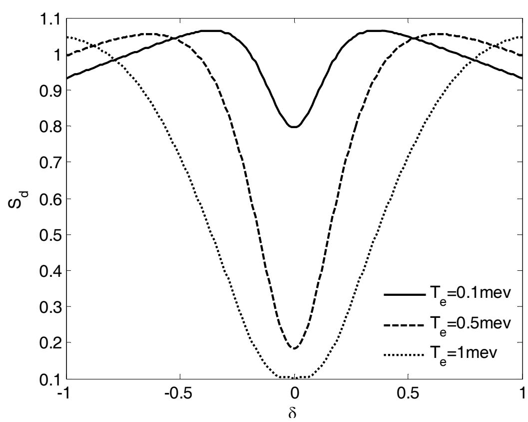

Fig. 5 Steady- state entanglement as a function of probe field detuning δ for three different values of T_e. Other parameters are $\gamma_{10} = 0.554meV$, $\gamma_{20} = 0.001meV$, $\Gamma_{12} = 2meV$, $\Gamma_{10} = 0.554meV$, $\Gamma_{20} = 0.005meV$, $\Omega = 0.5meV$, and $\omega_{12} = 0.2meV$.

We should emphasize that the tunneling coefficient depends on the gate voltage via relation [41]:

$$T_e(E,V) = \exp\left\{\left(-\frac{4l_b}{3eV}\right)\left(\frac{2m}{\hbar^2}\right)^{3/2}(V_0 - E)\right\}, \quad (9)$$

$(V_0 - E < E < V_0)$

which can be obtained by WKB approximation. We observe that $T_e(E,V)$ can be controlled by the gate voltage V. Here e and m are the electric charge and mass of electron, while l_b and V_0 are the barrier width and depth of the barrier.

Finally, we investigate the effect of spontaneous emission intensity on quantum entropy. In Fig. 6, we observe that quantum entropy increase by increasing the rate of spontaneous emission. However, for a strong spontaneous emission field the quantum entropy will decrease. Physically, strong spontaneous emission field may destroy the quantum coherence. In fact, decreasing the quantum coherence may decrease the quantum entropy.

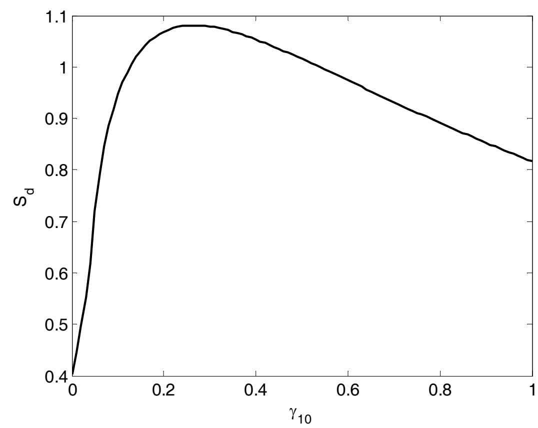

Fig. 6 Quantum entropy versus intensity of spontaneous emission field for the parameters $\delta = 0.1meV$, $\gamma_{20} = 0.1meV$, $\Gamma_{12} = 2meV$, $\Gamma_{10} = 0.554meV$, $\Gamma_{20} = 0.005meV$, $T_e = 0.5meV$, and $\omega_{12} = 0.2meV$.

V. Conclusion

In summary, we investigated the degree of entanglement of three- level atom QD molecule and its spontaneous emission fields via reduced quantum entropy. We found that the entanglement of QD molecule and its spontaneous emission fields increase by increasing the laser intensity. i.e. Rabi frequency. However, the degree of the QD molecule and its spontaneous emission fields decrease by increasing the gate voltage.

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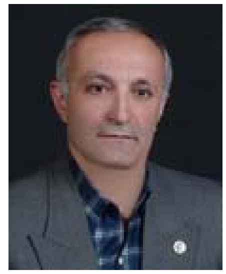

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