Control of the Nonclassical Properties of the Two-Mode Kerr Nonlinear Optical System Based on the Nonlinear Coherent States Approach

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ABSTRACT— One of the nonlinear optical phenomena which arise out of a $\chi^{(3)}$ nonlinearity, is the intensity dependence of the refractive index. In this paper, we describe this phenomenon based on the nonlinear coherent states approach. We show that the deformed two-dimensional oscillator algebra can be used to describe this nonlinear optical system. Then, we construct the nonlinear coherent states for this nonlinear optical system and study their quantum statistical properties. Finally, we find that by changing nonlinearity of the media, it is possible to control the nonclassical properties of the system.

KEYWORDS: Deformed oscillator algebra, Nonlinear coherent state, Nonlinear optical system.

I. INTRODUCTION

The effect of a nonlinear medium on the experimental outputs of the physical experiments is one of the issues that has been extensively considered and studied. Nonlinear optics is one of the areas used to investigate the nonlinear response of the media to different modes of the light field. Also, in recent years, the quantum nonlinear optics attracted more attentions [1, 2]. Some of the crucial quantum mechanically phenomena can be observed by quantization of electromagnetic field in nonlinear optics [3, 4]. By using nonlinear optical media together with systems exhibiting strong photon–photon interactions, one can find several unique applications, including control the light, single-photon transistors, and optical quantum logic media [1].

On the other hand, coherent states (CSs) of the harmonic oscillator as well as generalized CSs associated with various algebras play an important role in various theoretical and experimental fields of modern physics, including quantum optics and quantum communication. Among the generalized CSs, f-deformed CSs or nonlinear CSs (NCSs) have become a trend in recent years [5], mostly because they exhibit nonclassical properties such as amplitude squeezing and quantum interference [6]. In addition, NCSs have been able to describe the nonlinear aspects of some quantum phenomena, such as the optical Kerr effect [7].

Our aim in this paper is to describe the two-mode Kerr nonlinear optical system by using the NCSs approach and investigate its nonclassical properties. For this purpose, by using the deformed oscillator algebra, we find the corresponding deformation function, and construct the NCSs associated with this nonlinear system and then, we study their nonclassical properties.

The paper is organized as follows. In Section II, we describe the algebra of the two-mode Kerr nonlinear optical effect by using the f-deformed oscillator algebra. We also show that this algebra can be identified as a new type of deformed $su(2)$ algebra. Then, we construct the two-mode nonlinear CSs associated with our nonlinear optical system. Section III is devoted to the study of the quantum statistical properties of the constructed NCSs. Especially, the effect of nonlinearity of the medium on the nonclassical properties is clarified. In Section IV, we study the generation of the two-mode NCSs. A summary and concluding remarks are presented in Section V. Finally, their resolution of identify is provided in Appendix.

II. Nonlinear Coherent States Corresponding to Nonlinear System

As is known, the most common phenomenon to arise out of a nonlinearity is the intensity-dependent refractive index $\chi^{(3)}$. The Kerr and cross-Kerr effects are the two most common effects in nonlinear optics. In these effects, the refractive index of the medium consists of two terms: one term is constant and is the usual linear index of refraction while, the other one is proportional to the intensity of the field. Interaction of the above mentioned nonlinear optical system with a two-mode quantum field can be described by the following Hamiltonian:

$$\hat{H} = \hbar\omega_a \hat{a}^\dagger \hat{a} + \hbar\omega_b \hat{b}^\dagger \hat{b} + \hbar\chi_a \hat{a}^{\dagger 2}\hat{a}^2 + \hbar\chi_b \hat{b}^{\dagger 2}\hat{b}^2, \tag{1}$$

where $\hat{a}\left(\hat{a}^\dagger\right)$ and $\hat{b}\left(\hat{b}^\dagger\right)$ are the annihilation (creation) operators for the first and second modes, respectively. Also, χ_a and χ_b are proportional to the third order parameter of nonlinearity $\chi^{(3)}$ [8].

If we now compare the energy spectrum of the mentioned nonlinear system with the energy spectrum of a two-dimensional deformed oscillator Hamiltonian:

$$\hat{H} = \hbar\omega_a \hat{A}^\dagger \hat{A} + \hbar\omega_b \hat{B}^\dagger \hat{B}, \tag{2}$$

where, $\hat{A} = \hat{a}\hat{f}_a(\hat{n}_a)$ and $\hat{B} = \hat{b}\hat{f}_b(\hat{n}_b)$, we obtain the corresponding deformation functions as:

$$\hat{f}_i(\chi_i, \hat{n}_i) = \sqrt{1 + \frac{\chi_i}{\omega_i}\left(\hat{n}_i - 1\right)}, \quad i = a, b. \tag{3}$$

As it is seen, these deformation functions are functions of the number operators of the first and second mode, n_a and n_b, ω_a and ω_b, and the parameters of nonlinearity χ_a and χ_b. As will be shown later, it is possible to control the nonclassical properties by controlling these parameters.

In the following, we show that two-mode nonlinear optical system can also be described by a new type of deformed $su(2)$ algebra [9, 10]. We can construct a deformed $su(2)$ algebra corresponding to the two-mode Kerr medium, by defining a f-deformed (nonlinear) two-mode realization for its generators, in the similar approach of introducing the f-deformed oscillator algebra, as:

$$\begin{aligned} \hat{J}_+^{(\chi)} &= \hat{A}^\dagger \hat{B} = \hat{f}_a(\chi_a, \hat{n}_a)\hat{a}^\dagger \hat{b}\hat{f}_b(\chi_b, \hat{n}_b), \\ \hat{J}_-^{(\chi)} &= \hat{B}^\dagger \hat{A} = \hat{f}_b(\chi_b, \hat{n}_b)\hat{b}^\dagger \hat{a}\hat{f}_a(\chi_a, \hat{n}_a), \\ \hat{J}_0^{(\chi)} &= \frac{1}{2}\left(\hat{n}_a - \hat{n}_b\right). \end{aligned} \tag{4}$$

It can be easily shown that the above deformed operators satisfy the following deformed $su(2)$ algebra:

$$\begin{aligned} \left[\hat{J}_0^{(\chi)}, \hat{J}_\pm^{(\chi)}\right] &= \pm \hat{J}_\pm^{(\chi)}, \\ \left[\hat{J}_+^{(\chi)}, \hat{J}_-^{(\chi)}\right] &= 2\hat{J}_0^{(\chi)} + O(\chi). \end{aligned} \tag{5}$$

It is clear that the limit $\chi_a \to 0$ and $\chi_b \to 0$, the above $su_\chi(2)$ algebra reduces to the standard nondeformed $su(2)$ algebra. By using this nonlinear two-mode realization of the $su_\chi(2)$, we obtain:

$$\hat{J}_-^{(\chi)}\left|0, N\right\rangle = 0, \qquad \hat{J}_+^{(\chi)}\left|N, 0\right\rangle = 0. \tag{6}$$

The first equation states that $\left|0, N\right\rangle \equiv \left|0\right\rangle_a \otimes \left|N\right\rangle_b$ is the lowest weight state of the $su_\chi(2)$ algebra. It is also obvious from the second equation that, for each constant value of N, we encounter with a finite-dimensional Hilbert space. Now, we are intended to construct CSs associated with our nonlinear optical system. We can make use of the formalism of truncated CSs [11] and define two-mode Kerr nonlinear CSs (TMK-NCSs) corresponding to the nonlinear two-mode realization of the $su_{\chi}(2)$ algebra as:

$$\begin{aligned}\left|\mu\right\rangle_{\chi} &= C^{-1}\exp(\mu\hat{J}_{+}^{(\chi)})\left|0,N\right\rangle \\ &= C^{-1}\sum_{n=0}^{N}\sqrt{\binom{N}{n}}F_{a,b}(n)!\,\mu^{n}\left|n,N-n\right\rangle,\end{aligned} \tag{7}$$

where,

$$F_{a,b}(n)=\frac{f_a(\chi_a,n)}{f_b(\chi_b,N-n)}, \tag{8}$$

and C is the normalization constant. It is obvious that the coherent states $\left|\mu\right\rangle_{\chi}$ can be considered as a family of nonlinear coherent states corresponding to our nonlinear system.

It is clear that the limit $\chi_a\to 0$ and $\chi_b\to 0$, $F_{a,b}(n)\to 1$ and the above deformed TMK-NCSs reduce to the CSs for bosonic realization of the $su(2)$ algebra [12]. Also, in Appendix A, we explicitly show that TMK-NCSs form an ovecomplete set.

III. Quantum Statistical Properties of the TMK-NCSs

In the present section, we shall proceed to study some quantum statistical properties of the constructed CSs, including the cross-correlation, entanglement and Mandel parameter.

One of the interesting properties of the two-mode TMK-NCSs is the anticorrelation between the modes. This anticorrelation is defined by the normalized cross-correlation function as [13]:

$$g^{(2)}(0)=\frac{\left\langle\hat{n}_a\hat{n}_b\right\rangle}{\left\langle\hat{n}_a\right\rangle\left\langle\hat{n}_b\right\rangle}. \tag{9}$$

In figure 1, we have plotted the cross-correlation function with respect to μ for different values of $\kappa_b=\dfrac{\chi_b}{\omega_b}$.

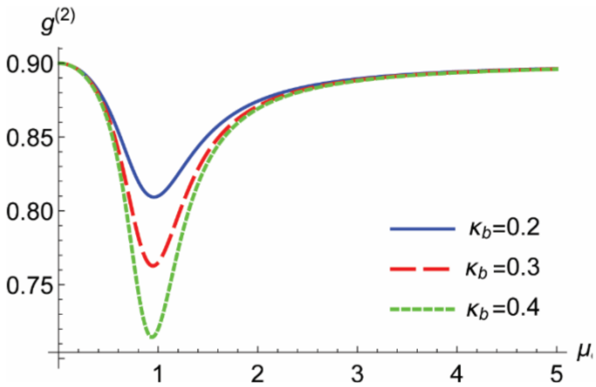

Fig. 1. Cross-correlation function for the state $\left|\mu\right\rangle_{\chi}$ versus μ for $\kappa_b=0.2$ (solid blue line), $\kappa_b=0.3$ (dashed red line), $\kappa_b=0.4$ (dotted green line), with $N=10$.

It is clear that by increasing the μ, the anticorrelation between two modes increases and then decreases. Also, we see that for a fixed μ, by increasing κ_b, the anticorrelation between two modes is increased. Furthermore, it is seen that for any values of μ, the cross-correlation function is less than one. This indicates that the two discussed modes are anticorrelated. Physically this means that there is no tendency for photons in the different modes to be created or annihilated simultaneously, if we perform a jointly detection.

The von Neumann entropy is a measure of entanglement for pure bipartite states [14]. This measure becomes $\ln(N+1)$ when an $(N+1)$-dimensional bipartite system is maximally entangled. The von Neumann entropy $S(\hat{\rho}_a)$ for the reduced density operator $\hat{\rho}_a$, is:

$$S(\hat{\rho}_a)=-Tr_a[\hat{\rho}_a\ln\hat{\rho}_a]. \tag{10}$$

Therefore, by using the $S(\hat{\rho}_a)$, we can calculate entanglement of the two modes of the TMK-NCSs, [**Eq. (7)**].

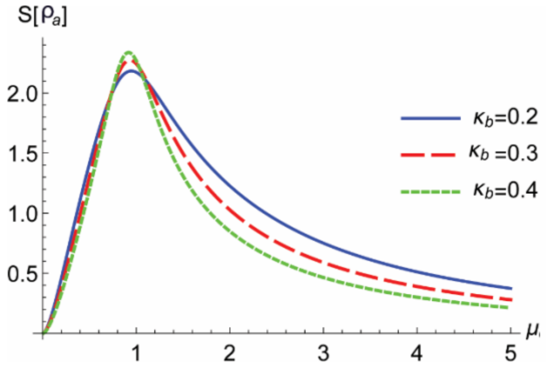

Fig. 2. The von Neumann entropy $S(\hat{\rho}_a)$ versus μ for $\kappa_b = 0.2$ (solid blue line), $\kappa_b = 0.3$ (dashed red line), $\kappa_b = 0.4$ (dotted green line), with $N = 10$.

Figure 2 shows the von Neumann entropy $S(\hat{\rho}_a)$ with respect to μ for different values of κ_b. As it is seen, the entropy increases to a maximum then decreases and declines slowly to zero. It is also seen that by increasing the κ_b, the TMK-NCSs can give us higher entanglement and also, the maximum of entanglement occurs in smaller values of μ.

In order to determine the quantum statistics of the TMK-NCSs, we consider Mandel parameter [15] for two modes of $|\mu\rangle_\chi$ as

$$Q_i = \frac{(\Delta n_i)^2 - \langle \hat{n}_i \rangle}{\langle \hat{n}_i \rangle}, \quad i = a,b \tag{11}$$

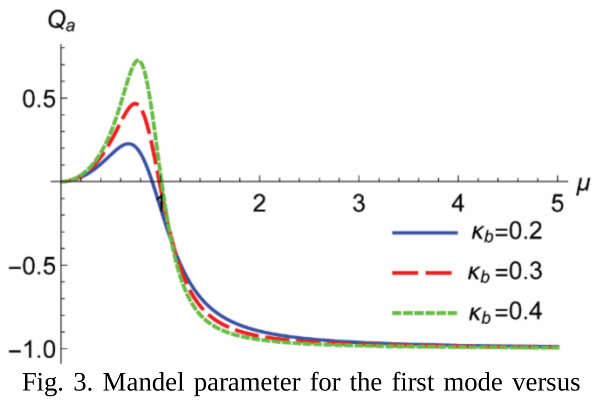

Fig. 3. Mandel parameter for the first mode versus μ for $\kappa_b = 0.2$ (solid blue line), $\kappa_b = 0.3$ (dashed red line), $\kappa_b = 0.4$ (dotted green line), with $N = 10$.

Negative values of Q correspond to states whose photon variance is less than mean (sub-Poissonian and photon antibunching). Q is positive for a super-Poissonian distribution (photon bunching), and $Q = 0$ corresponds to Poissonian distribution.

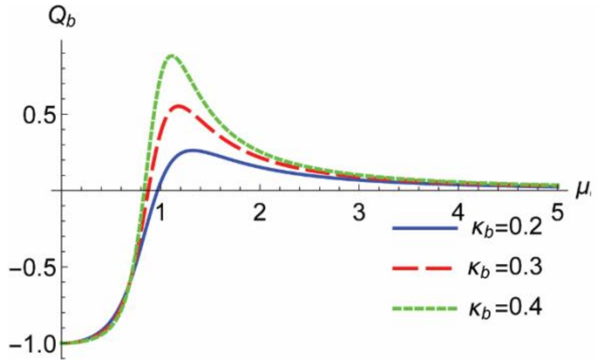

Fig. 4. Mandel parameter for the second mode versus μ for $\kappa_b = 0.2$ (solid blue line), $\kappa_b = 0.3$ (dashed red line), $\kappa_b = 0.4$ (dotted green line), with $N = 10$.

Figures 3 and 4 show the effect of nonlinearity on the variation of Mandel parameters of two modes of our nonlinear CSs with $N = 10$ for different values of κ_b. As it is seen, the first mode has super-Poissonian statistics and by increasing μ, the photon-counting statistics of the first mode of the CSs tend to sub-Poissonian. On the other hand, the second mode has sub-Poissonian statistics and by increasing μ, the photon-counting statistics of this mode of the CSs tend to super-Poissonian and finally it reaches to Poissonian statistics. In addition, for a fixed parameter μ, by increasing nonlinearity κ_b, the Mandel parameter is increased. In the following, by considering the behaviour of Mandel parameter due to increasing the amplitude of the coherent state, μ, i.e., increasing nonclassical properties in mode a and decreasing nonclassical properties in mode b, we can define critical parameters for the transition between classical and nonclassical properties. So that for the mode $a(b)$ at the critical value μ_{c_a} (μ_{c_b}), the super-Poissonian (sub-Poissonian) statistics of the CSs $|\mu\rangle_\chi$ is converted to the sub-Poissonian (super-Poissonian) statistics.

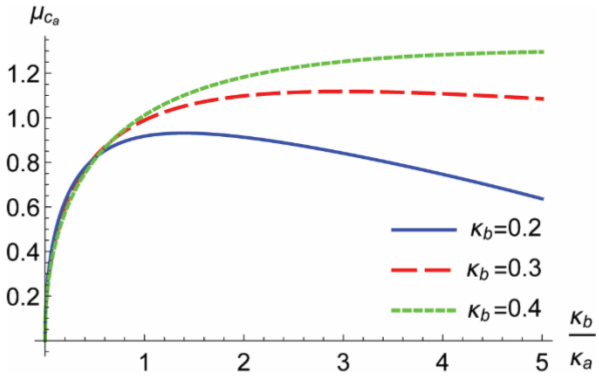

Fig. 5. μ_{c_a} mode a versus L for $\kappa_b = 0.2$ (solid blue line), $\kappa_b = 0.3$ (dashed red line), $\kappa_b = 0.4$ (dotted green line), with $N = 10$.

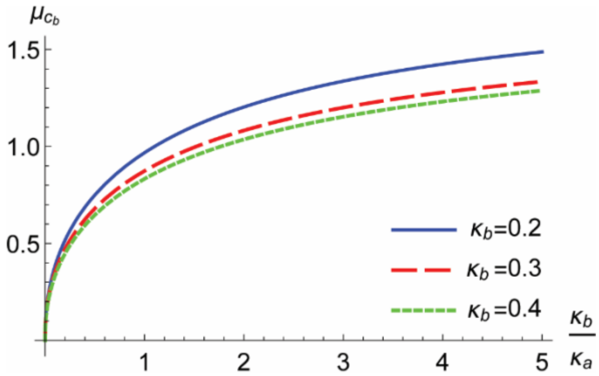

Fig. 6. μ_{c_b} mode b versus L for $\kappa_b = 0.2$ (solid blue line), $\kappa_b = 0.3$ (dashed red line), $\kappa_b = 0.4$ (dotted green line), with $N = 10$.

In Figs.5 and 6, we have plotted the critical parameters μ_{c_a} and μ_{c_b} with respect dimensional parameter $L \equiv \kappa_b / \kappa_a$ for different values of κ_b with $N = 10$. As it is seen, by increasing L, the parameter μ_{c_a} first increases and then decreases to a lower level. On the other hand, the critical μ_{c_b} is increased by increasing L. Also, for a fixed value of L, by increasing κ_b, μ_{c_a} (μ_{c_b}) is increased (decreased). In other words, by changing the ratio of the nonlinear parameters, we can control the nonclassical properties of the two modes. For instance, by increasing the parameter L, it is possible to prepare the system in the coherent state $|\mu\rangle_\chi$ with larger values of μ, for which, the first mode still shows nonclassical properties.

IV. Generation

A scheme for the generation of a class of two-mode field states in a cavity by the transfer of atomic coherence to the cavity field was proposed in **[16]**. They studied a Raman-coupled three-level atomic system interacting with a two-mode field in a cavity. Because of the special property of this interaction, the total number of photons in both modes is a constant quantity. The atomic system consists of a series of three-level $\Lambda -$type atoms initially prepared in a linear superposition of their two nondegenerate ground states as $|3\rangle + \varepsilon_l |1\rangle$, where ε_l is the complex amplitude coefficient for the l th atom and $E_3 < E_1$. These atoms interact with a two-mode cavity field which is initially prepared its first mode in a vacuum state, and the second mode contains N photons. Each injected atom increases the photon number by one in the first mode by destroying one photon in the second mode. It is assumed that after the passage of the $(l-1)$ th atom and just before the injection of the l th atom, the cavity field is in a state $\left|\phi^{(l-1)}\right\rangle = \sum_{n=0}^{N} \phi_n^{(l-1)} |n, N-n\rangle$.

As soon as an atom exits the cavity one detects whether the atom is in the state $|3\rangle$ or $|1\rangle$. If the atom is in the ground state, then one should continue the process for more energy transfer from the atom to the field to achieve the desired state. Otherwise, if the atom is found in the excited state then one has to repeat the process. It is worth noting that the new coefficients $\phi_n^{(l)}$ of the field state after the exit of the l th atom, i.e., $\left|\phi^{(l)}\right\rangle = \sum_{n=0}^{N} \phi_n^{(l)} |n, N-n\rangle$, are given in terms of the old coefficients $\phi_n^{(l-1)}$ according to a recurrence relation [16].

Using a similar approach, we can prepare the TMK-NCSs, [**Eq. (7)**], which have a constant total number of quanta of the two modes N. For this purpose, one has to find that the combination $\left|\phi^{(N-1)}\right\rangle = \sum_{n=0}^{N} \phi_n^{(N-1)} |n, N-n\rangle$ of

N number states, which yields $|\mu\rangle_\chi$ after the N th atom prepared in an appropriate internal state $|3\rangle + \varepsilon_l(\chi)|1\rangle$, has passed through the cavity and has been detected in the ground state. We can construct a characteristic polynomial equation for $\varepsilon_l(\chi)$ of order N and choose the lowest value of $\varepsilon_N(\chi)$ out of N roots based on [17]. Having achieved $\varepsilon_N(\chi)$, we obtain a set of $\phi_n^{(N-1)}$'s. In the next step we take $|\phi^{(N-1)}\rangle$ as a new desired state, which one has to obtain by sending $N-1$ atoms through the cavity. For the state $|\phi^{(N-1)}\rangle$, the same calculations as for the state $|\mu\rangle_\chi$ may be done to achieve the parameter $\varepsilon_{N-1}(\chi)$ and state $|\phi^{(N-1)}\rangle$ with $N-1$ coefficients $\phi_n^{(N-2)}$. One repeats the calculations until one ends up with the initial field state. A string of χ – dependent complex numbers $\varepsilon_1(\chi), \varepsilon_2(\chi), ..., \varepsilon_N(\chi)$ defines the internal states of a sequence of N atoms one should inject into the cavity in order to obtain the desired two-mode Kerr NCSs in a two-mode resonator.

V. CONCLUSION

In this paper, to investigate the quantum approach to nonlinear optics (QNO) we have searched for a relation between the deformation function of the f-deformed oscillator algebra and a two-mode Kerr nonlinear optical medium. We found that it is possible to describe the two-mode Kerr system as a deformed two dimensional harmonic oscillator algebra as well as a deformed $su(2)$ algebra. We have constructed the NCSs corresponding to this two-mode Kerr medium and studied their quantum statistical properties. Also, we have shown that by changing nonlinearity of the media and μ, the quantum statistical properties of the TMK-NCSs can be controlled. In addition, by introducing the critical parameters μ_{c_a} and μ_{c_b} as transition points from nonclassical properties of the system to classical properties and by examining the nonlinear effects of the medium on the μ_{c_a} and the μ_{c_b}, we have shown that by changing the nonlinearity of the media, the nonclassical properties of both modes can be controlled. Finally, we have proposed a scheme to prepare the TMK-NCSs on a cavity.

APPENDIX A: RESOLUTION OF IDENTITY

In this appendix, we show that the TMK-NCSs form an overcomplete set. Since it is necessary to include a measure function $m\left(|\mu|^2\right)$ in the integral, we require

$$\int d^2\mu |\mu\rangle m\left(|\mu|^2\right)\langle\mu| = \sum_{n=0}^{N} |n\rangle\langle n| = \hat{\mathrm{I}}. \tag{A1}$$

In the case of $\chi_a = \chi_b = 0$ (we call it $\chi = 0$)

$$\begin{aligned}
&\int d^2\mu |\mu\rangle_{\chi=0} m_{\chi=0}(|\mu|^2)\ {}_{\chi=0}\langle\mu| \\
&= \sum_{n=0}^{N} \binom{N}{n} |n, n-N\rangle\langle n, n-N| \\
&\times \int d\left(|\mu|^2\right) d\theta \frac{|\mu|^2}{(1+|\mu|^2)^N} m_{\chi=0}\left(|\mu|^2\right) \\
&= \pi \sum_{n=0}^{N} \binom{N}{n} |n, n-N\rangle\langle n, n-N| \\
&\times \int_0^\infty d\left(|\mu|^2\right) d\theta \frac{|\mu|^2}{(1+|\mu|^2)^N} m_{\chi=0}\left(|\mu|^2\right).
\end{aligned} \tag{A2}$$

Thus, we should have

$$\int_0^\infty d\left(|\mu|^2\right) d\theta \frac{|\mu|^2}{\left(1+|\mu|^2\right)^N} m_{\chi=0}\left(|\mu|^2\right) = \frac{1}{\pi \binom{N}{n}}. \tag{A3}$$

The suitable choice for the measure function reads [6]

$$m_{\chi=0}\left(|\mu|^2\right) = \frac{N+1}{\pi} \frac{|\mu|^2}{\left(1+|\mu|^2\right)^2}, \tag{A4}$$

so, the resolution of identify is

$$\frac{N+1}{\pi}\int d\left(|\mu|^2\right)^2 |\mu\rangle_{\chi=0\ \chi=0}\langle\mu| = \hat{I} \qquad \text{(A5)}$$

In order to examine the resolution of identity for TMK-NCSs, we first define the deformed Binomial expansion [6]

$$\left(1+x\right)_\chi^N = \sum_{n=0}^{N}\binom{N}{n}_\chi x^n, \qquad \text{(A6)}$$

where

$$\binom{N}{n}_\chi = \binom{N}{n}\left\{\left[F_{a,b}(n)\right]!\right\}^2. \qquad \text{(A7)}$$

We see that when $\chi_a \to 0,\ \chi_b \to 0,$ $F_{a,b}(n) \to 1$ and deformed Binomial expansion becomes the well-known Binomial expansion. Now, using this definition we can rewrite the TMK-NCSs as,

$$|\mu\rangle_\chi = \left(1+|\mu|^2\right)_\chi^{-N/2}\sum_{n=0}^{N}\sqrt{\binom{N}{n}_\chi}\mu^n\,|n, N-n\rangle. \qquad \text{(A8)}$$

For the resolution of identify we should have

$$\int_{(\chi)} d^2\mu\,|\mu\rangle_\chi m_\chi\left(|\mu|^2\right)\,_\chi\langle\mu| = \sum_{n=0}^{N}|n, N-n\rangle\langle n, N-n| = \hat{\mathrm{I}}, \qquad \text{(A9)}$$

or, equivalently,

$$\pi\sum_{n=0}^{N}\binom{N}{n}_\chi |n-N\rangle\langle n-N| \times\int_0^\infty d\left(|\mu)|^2\right)d\theta\frac{|\mu|^2}{\left(1+|\mu|^2\right)_\chi^N} m_\chi\left(|\mu|^2\right) = \hat{I}. \qquad \text{(A10)}$$

If we define the following measure,

$$m_\chi\left(|\mu|^2\right) = \frac{N+1}{\pi}\frac{1}{\left(1+|\mu|^2\right)_\chi^2}, \qquad \text{(A11)}$$

and the following deformed version of Eq. (A5)

$$\int_{0(\chi)}^\infty d\left(|\mu|^2\right)^2\frac{|\mu|^{2n}}{\left(1+|\mu|^2\right)_\chi^2}|\mu\rangle_{\chi\ \chi}\langle\mu| = \frac{1}{\pi\binom{N}{n}_\chi}, \qquad \text{(A12)}$$

we arrive at the resolution of identify for the $|\mu\rangle_\chi$ as:

$$\frac{N+1}{\pi}\int_{(\chi)}\frac{d^2\mu}{\left(1+|\mu|^2\right)_\chi^2}|\mu\rangle_{\chi\ \chi}\langle\mu| = \hat{\mathrm{I}}. \qquad \text{(A13)}$$

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